1. Derive identities for $\sin 3\theta$ and $\cos 3\theta$ by calculating $e^{3i\theta} = (e^{i\theta})^3$.

2. Show that if $z$ and $w$ are complex numbers then $|zw| = |z||w|$.

3. If $z = a + ib$ and $w = c + id$ are complex numbers with $a, b, c, d$ real, derive a formula for the real and imaginary parts of $\frac{z}{w}$.

4. Let $g(x)$ be a real valued function defined everywhere on the real line. Suppose that there are only ten points $x_1, x_2, \ldots, x_{10}$ at which $g$ vanishes. That is, $g(x) = 0$ if and only if $x$ is one of $x_1, \ldots, x_{10}$ Let $G(x)$ be an antiderivative for $g$. That is, suppose

$$G'(x) = g(x),$$

at every $x$. What is the largest number of possible real values $x$ where $G(x)$ vanishes.

5. Let

$$f(x) = \sum_{j=1}^{\infty} a_j x^j,$$

be a power series and let $R$ be its radius of convergence. Let $R' < R$. Show that for every $\epsilon > 0$ there is $N(\epsilon)$ so that

$$|f(x) - \sum_{j=1}^{N} a_j x^j| < \epsilon,$$

for every $x$ with $|x| < R'$.

6. Let $n$ be a natural numbers. Show using the definition of integrability that $f(x) = x^n$ is integrable on any interval $[a, b]$ and that

$$\int_{a}^{b} f(x) dx = \frac{1}{n+1} (b^{n+1} - a^{n+1}).$$

Hint: The point of this problem is to practice using the definition of the integral.

7. Let $f$ and $g$ be integrable functions on $[a, b]$ show that $f + g$ is integrable on $[a, b]$.

8. Let $f$ be an bounded function on $[a, b]$. Let $K$ be the difference between the upper integral of $f$ on $[a, b]$ and the lower integral of $f$ on $[a, b]$. (Note that $f$ being integrable on $[a, b]$ means $K = 0$.) Show that for any $\epsilon > 0$ there is a partition $P$ of $[a, b]$ so that

$$U(f, P) - L(f, P) < K + \epsilon.$$