Review Problems: Math 1a Midterm 2014

These aren’t problems that will be on the midterm. These aren’t problems you have to turn in. These are problems that might help you if you study them.

1. Use induction to prove for $r \neq 1$

$$\sum_{j=1}^{n} j r^j = \frac{n r^{n+1}}{r - 1} - \left( \frac{r^{n+1} - r}{(r - 1)^2} \right).$$

2. Use induction to prove that for the Fibonacci sequence given by $f_1 = 1$, $f_2 = 1$ and

$$f_j = f_{j-1} + f_{j-2},$$

for $j \geq 2$ that we have

$$f_n = \frac{1}{\sqrt{5}} \left( \left( \frac{1 + \sqrt{5}}{2} \right)^n - \left( \frac{1 - \sqrt{5}}{2} \right)^n \right).$$

3. Show that if $\{a_n\}$ is a Cauchy sequence, it has a subsequence $\{a_{n_k}\}$ so that the series

$$\sum_{k=1}^{\infty} a_{n_{k+1}} - a_{n_k},$$

converges absolutely.

4. Determine whether or not the following series converges:

$$\sum_{n=1}^{\infty} \frac{\sqrt{n + 2014} - \sqrt{n}}{n}.$$ 

5. Calculate

$$\lim_{n \to \infty} \frac{4n^3 + 17n^2 + 12n + 27}{n^3 + 6n + 1997}.$$ 

Prove your answer using the definition of the limit.

6. Calculate

$$\lim_{u \to 2} \frac{\sqrt{4u + 1} - 3}{u - 2}.$$ 

Justify your answer, either using the definition of the limit or using the limit laws.

7. We say that a sequence $\{x_n\}$ is nondecreasing if $x_j \leq x_k$ whenever $k > j$. We say a sequence $\{x_n\}$ is nonincreasing if $x_j \leq x_k$ whenever $k < j$. Show that any sequence of real numbers has a subsequence which is either nondecreasing or nonincreasing.

8. Let $f(x)$ and $g(x)$ be functions from the real numbers to the real numbers which are continuous for every real $x$. Show that $f(g(x))$ is a function from the real numbers to the real numbers which is continuous at every real $x$. 

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