1 Problem 8.1.6

Verify Green’s theorem for the indicated region $D$ and boundary $\partial D$, and functions $P$ and $Q$:

$D = [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$, \quad $P(x,y) = \sin x$, \quad $Q(x,y) = \cos y$

2 Problem 8.1.13

Find the area bounded by one arc of the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$, where $a > 0$, and $0 \leq \theta \leq 2\pi$, and the $x$ axis (use Green’s theorem).

3 Problem 8.1.15

Evaluate the line integral

$$\int_C (2x^3 - y^3) \, dx + (x^3 + y^3) \, dy$$

where $C$ is the unit circle, and verify Green’s theorem for this case.

4 Problem 8.1.20

Let $P(x,y) = -y/(x^2 + y^2)$ and $Q(x,y) = x/(x^2 + y^2)$. Assuming $D$ is the unit disc, investigate why Green’s theorem fails for this $P$ and $Q$.  

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5 Problem 8.2.4

\[ S = \{(x, y, z) : x^2 + y^2 + z^2 = 1, \ z \geq 0\}, \ \partial S = \{(x, y, 0) : x^2 + y^2 = 1\}, \]
\[ F(x, y, z) = yi + zj + xk. \]
Verify Stokes’ theorem for \( S, \partial S \) and \( F \) where \( S \) is oriented as a graph. (Note that the book says \( \partial S = \{(x, y) : x^2 + y^2 = 1\} \) - which is wrong, since \( \partial S \subset \mathbb{R}^3 \) not \( \mathbb{R}^2 \))

6 Problem 8.2.5

\[ S = \{(x, y, z) : z = 1 - x^2 - y^2, \ z \geq 0\}, \ \partial S = \{(x, y, 0) : x^2 + y^2 = 1\}, \]
\[ F(x, y, z) = zi + xj + (2zx + 2xy)k. \]
Verify Stokes’ theorem for \( S, \partial S \) and \( F \) where \( S \) is oriented as a graph. (Note that the book says \( \partial S = \{(x, y) : x^2 + y^2 = 1\} \) - which is wrong, since \( \partial S \subset \mathbb{R}^3 \) not \( \mathbb{R}^2 \))

7 Problem 8.2.8

Let \( C \) be the closed, piecewise smooth curve formed by traveling in straight lines between the points \((0, 0, 0), (2, 1, 5), (1, 1, 3), \) and back to the origin, in that order. Use Stokes’ theorem to evaluate \( \int_C xzy \ dx + xy \ dy + x \ dz. \)

8 Problem 8.2.17.

Calculate the surface integral \( \iint_S (\nabla \times F) \cdot dS \), where \( S \) is the hemisphere \( x^2 + y^2 + z^2 = 1, \ x \geq 0, \) with the upward orientation, and \( F = x^3i - y^3j. \)