In the sequel, $V$ denotes a vector space defined over the field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ unless otherwise specified.

Problem 1. Read from the textbook: Chapter 2, Sections 1–4.

Problem 2 (10pts). From the textbook. Ch. 1, Problem 6.6.

Problem 3 (20pts). Let $T : \mathbb{F}^4 \rightarrow \mathbb{F}^4$ be the linear map whose matrix with respect to the standard basis of $\mathbb{F}^4$ is

$$A = \begin{bmatrix}
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}. $$

Compute $T^{2015}v$, where $v = (0, 1, 1, 2)^T$.

Problem 4 (30pts). From the textbook. Ch. 2, Problem 2.1 (a) (e). (Note you need to solve the systems.)

Problem 5 (20pts). From the textbook. Ch. 2, Problem 3.1.

Problem 6 (20pts). Let $a, b, c \in \mathbb{F}$ be three distinct numbers in $\mathbb{F}$. Show that

$$(1, a, a^2)^T, (1, b, b^2)^T, (1, c, c^2)^T$$

are three linearly independent vectors in $\mathbb{F}^3$. 