MATH 109B-HOMEWORK SET 4

The homework set is due Thursday 2/26 at 2pm.


2. Prove that a surface of rotation can always be locally parametrized in such a way that the new parametrization is angle preserving.

3. Let $\beta$ be a Frenet curve in $\mathbb{R}^3$ and let $D = \tau T + \kappa B$ (so called the Darboux vector). Show that the ruled surface this defines, given by
   $$f(u, v) = \beta(u) + vD(u),$$
   is a developable surface.

4. Suppose we are given a surface element with $K < 0$. Show that this surface is a minimal surface if and only if the asymptotic curves at each point are perpendicular to one another.

5. Prove that there are no compact (i.e., bounded and closed in $\mathbb{R}^3$) minimal surfaces.

6. Determine the umbilic points of the elipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

7. Show that if a surface is tangent to a plane along a curve, then the points of this curve are either parabolic or level points (planar).