MATH 109B-HOMEWORK SET 3

The homework set is due Thursday 1/29 at 2 pm.

1. Let $\beta$ be a unit-speed curve in $\mathbb{R}^3$ with $\kappa > 0$, and suppose that $E_1, E_2, E_3$ is a frame field on $\mathbb{R}^3$ such that the restriction of these vector fields to $\beta$ gives the Frenet-frame field $T, N, B$ of $\beta$. Prove that

$$\omega_{12}(T) = \kappa, \omega_{13}(T) = 0, \omega_{23}(T) = \tau.$$ 

Then deduce the Frenet formulas from the connection equations.

2. Show that the osculating cubic parabola of a Frenet curve $\beta$ in $\mathbb{R}^3$, defined by

$$s \to \beta(0) + sT(0) + \frac{s^2}{2}\kappa(0)N(0) + \frac{s^3}{6}\kappa(0)\tau(0)B(0)$$

has, at the point $s = 0$, the same curvature $\kappa(0)$ and torsion $\tau(0)$ as $\beta$ itself.

3. Construct a non-planar curve which is a Frenet curve except for a single point and in the complement of this point satisfies $\tau \equiv 0$. 