In the sequel, $V$ denotes a vector space defined over the field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ unless otherwise specified.

1. Read from the textbook: Chapter 5, Section 4-6, Chapter 6, Section 1-2.

2. [20pts] From the textbook: Chapter 5, Problem 4.4.

3. [20pts] From the textbook: Chapter 5, Problem 6.1 (only first and third matrices).

4. [40pts] Let $v_1 = (1, 0, 1)^T$, $v_2 = (2, 2, 0)^T$, and $V$ be the subspace in $\mathbb{R}^3$ spanned by $v_1$ and $v_2$.
   (a) Find $\{u_1, u_2\}$ an orthogonal basis of $V$.
   (b) For $i = 1, 2$, express $v_i$ as a linear combination of the new basis.
   (c) Compute the orthogonal complement of $V$ in $\mathbb{R}^3$.
   (d) Complete $\{u_1, u_2\}$ to an orthogonal basis of $\mathbb{R}^3$.
   (e) Let $w_1 = (1, -1, -1)^T$, $w_2 = (1, 1, 1)^T$. For each of $w_i$, determine whether $w_i$ belong to the space $V$. If possible, write $w_i$ as a linear combination of $v_1, v_2$. If not, find the distance from $w_i$ to $V$.
   (f) For $w = (3023, 2345, 678)^T$: does $w$ belong to the space $V$? (Hint: do not compute the distance!)

5. [20pts] Let $T : V \to V$ be a self-adjoint linear operator on a real vector space $V$. Assume that $\langle Tv, v \rangle \geq 0$ for every $v \in V$. Show that for every positive integer $k$, there is a linear operator $S : V \to V$ such that $T = S^k$. 