In the sequel, $V$ denotes a vector space defined over the field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ unless otherwise specified.

1. Read from the textbook: Chapter 1, Sections 5-6, Chapter 2, Section 1-2, 4.

2. [10pts] From the textbook. Ch. 1, Problem 6.9.

3. [20pts] Let $T : \mathbb{F}^4 \to \mathbb{F}^4$ be the linear map whose matrix with respect to the standard basis of $\mathbb{F}^4$ is

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$ 

Compute $T^{2015}v$, where $v = (0, 1, 1, 2)^T$.

4. [30pts] From the textbook. Ch. 2, Problem 2.1 (a) (c). (Note you need to solve the systems.)

5. [20pts] Find a basis of each of the following vector space

(1) $\{(x_1, x_2, \ldots, x_n)^T \in \mathbb{F}^n \mid x_1 + x_2 + \cdots + x_n = nx_n\}$.

(2) $\{(x_1, x_2, x_3, x_4, x_5)^T \in \mathbb{F}^5 \mid x_1 = 3x_2, x_3 = 7x_4, x_1 + x_2 + x_5 = 0\}$.

6. [20pts] Let $a, b, c \in \mathbb{F}$ be three distinct numbers in $\mathbb{F}$. Show that $(1, a, a^2)^T, (1, b, b^2)^T, (1, c, c^2)^T$ are three linearly independent vectors in $\mathbb{F}^3$. 
