In the sequel, $V$ denotes a vector space defined over the field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ unless otherwise specified.

1. Read from the textbook: Chapter 1, Sections 1-7.

2. [40pts] For each of the following subset of $\mathbb{F}^n$, $n \geq 2$, determine whether it is a subspace of $\mathbb{F}^n$. Explain the reason.
   (1) $\{(x_1, x_2, \ldots, x_n)^T \in \mathbb{F}^n \mid x_1 + x_2 + \cdots + x_n = n\}$.
   (2) $\{(x_1, x_2, \ldots, x_n)^T \in \mathbb{F}^n \mid x_1 + x_2 + \cdots + x_n = nx_n\}$.
   (3) $\{(x_1, x_2, \ldots, x_n)^T \in \mathbb{F}^n \mid x_1^2 + x_2^2 + \cdots + x_n^2 = 0\}$. (You need separate the case $\mathbb{F} = \mathbb{R}$ or $\mathbb{F} = \mathbb{C}$.)
   (4) $\{(x_1, x_2, x_3, x_4, x_5)^T \in \mathbb{R}^5 \mid x_1 = 3x_2, x_3 = 7x_4, x_1 + x_2 + x_5 = 0\}$.

3. [15pts] From the textbook. Ch. 1 Problem 3.1 (a), (b), (c).

4. [10pts] From the textbook. Ch. 1 Problem 3.3 (a), (d).

5. [15pts] From the textbook. Ch. 1 Problem 5.5.

6 [20pts] Can the vectors $v_1, v_2$ be extended to a basis of $\mathbb{F}^3$? If yes, write down such a basis (there are many possible answers). Justify your answer.
   (1) $v_1 = (1, 0, 2)^T$ and $v_2 = (2, 0, 4)^T$;
   (2) $v_1 = (1, 0, 2)^T$ and $v_2 = (2, 1, 4)^T$. 
