Home Work Set 2

- Due date is 9 am, October 30 in 106 ANB (before class).
- HW2 has four problems. Collaborations are allowed but write your own solution.

Problem 1. (do Carmo, p. 86) (a) Prove that if $G$ is the geodesic field on $TM$ then $\text{div}G = 0$.
(b) Conclude from this that the geodesic flow preserves the volume of $TM$.
(see Hint in do Carmo’s book)

Problem 2. (Lee, p. 87) Let $M \subset \mathbb{R}^3$ be a surface of revolution, parameterized as in Exercise 3-3 (Lee, p.25) $\varphi(\theta, t) = (a(t)\cos\theta, a(t)\sin\theta, b(t))$.
It will simplify the computations if we assume that the curve $\gamma$ is a unit speed.
- (a) Compute the Christoffel symbols of the induced metric in $(\theta, t)$ coordinates.
- (b) Show that each ”meridian” $\{\theta = \theta_0\}$ is a geodesic on $M$.
- (c) Determine necessary and sufficient condition for a ”latitude circle” $\{t = t_0\}$ to be a geodesic.

Problem 3. (Lee, p.87) Let $H^n_R$ denote the n-dimensional hyperbolic space of radius $R$.
- (a) Determine the unit speed parametrization of the geodesic in the hyperboloid model starting at $N = (0, \ldots, R)$ with initial
tangent vector $\partial/\partial \xi^1$.
- (b) Prove that each geodesic on $H^n_R$ is defined for all $t \in \mathbb{R}$ and
that the image of each geodesic is an entire branch of a great hyperbola.

Problem 4. (Lee, p.113) Suppose $\bar{M}$ and $M$ are Riemannian manifolds, and $p : \bar{M} \to M$ is a smooth covering map that also a local isometry. If either $M$ or $\bar{M}$ is complete, show that the other is also.