

A. S. Kechris: Global Aspects of Ergodic Group Actions; Corrections and Updates (November 26, 2016)

Page x, line 3: Replace “the equivalence” by “an ergodic equivalence”.

Page 8, last 7 lines of the proof of 2.5: Replace ξ_i by η_i .

Page 12, line 3-: Add “.” at the end.

Page 19, Theorem 3.13: This was originally proved in H.A. Dye, On groups of measure preserving transformations, I, *Amer. J. Math.*, **81(1)** (1959), 119–159.

Page 20, Theorem 4.1: This was originally proved in H.A. Dye, On groups of measure preserving transformations, II, *Amer. J. Math.*, **85(4)** (1963), 551–576.

Page 22, line 20-: François Le Maître pointed out that in [BG1] the authors prove this result for the class of measure-class preserving equivalence relations but the older result for the case of measure preserving equivalence relations is due to H.A. Dye, On groups of measure preserving transformations, II, *Amer. J. Math.*, **85** (1963), 551–576, Proposition 5.1.

Page 26: An update concerning the results in the paragraph following 4.12:

Matui has shown that for ergodic hyperfinite E , $t([E]) = 2$, and $t([E_n]) \leq 2(n + 1)$. Independently Marks also proved that $t([E]) = 2$ and moreover showed that $t([E_n]) \leq 2n$ and if E_n is induced by a free modular action of F_n , then $t([E_n]) = n + 1$.

Finally Le Maître proved that for *every ergodic* measure preserving E one has the formula $t([E]) = \lfloor C_\mu(E) \rfloor + 1$, where $\lfloor C_\mu(E) \rfloor$ is the integer part of the cost of E , and therefore $t([E_n]) = n + 1$. He also obtained a related formula when E is aperiodic but not necessarily ergodic.

Page 30, line 1-: After “3.14” add “and 2.6”.

Page 67, line 18: Delete “ergodic”.

Page 71, 10.9: Add in the statement of the theorem that Γ is an infinite countable group.

Page 71, line 4: Before “Let” add “By 10.8, we can assume that a is free”.

Page 71, line 11: Replace “The extreme ... Krein-Milman” by “Using the ergodic decomposition of a and the fact that a is free, we see that”.

Page 71, line 13: After “ergodic,” add “and non-atomic,”.

Page 73, line 1-: Replace “ $\Gamma \leq Z(\Delta)$ ” by “ $\Gamma \leq C_\Delta(T)$ = the centralizer of T in Δ ”.

Page 75, Remark: Robin Tucker-Drob answered the question at the end of this remark by showing that $C^*(\Gamma) = C^{**}(\Gamma)$ for all infinite groups Γ . The proof is as follows:

It is enough to show that if Γ has property (T), then $C(\Gamma) = C^*(\Gamma) = C^{**}(\Gamma)$. Indeed, suppose that Γ has property (T). Then Γ is finitely generated (see [BdlHV, 1.3.1]), so by Corollary 10.14 we have $C(\Gamma) = C^*(\Gamma)$. It remains to show that $C(\Gamma) = C^{**}(\Gamma)$. By Theorem 13.1 the set A , consisting of all actions $a \in \text{FRERG}(\Gamma, X, \mu)$ with weakly dense conjugacy class in $\text{FRERG}(\Gamma, X, \mu)$, is dense G_δ in $(\text{FRERG}(\Gamma, X, \mu), w)$. Fix $a \in A$. Let $b \in \text{FR}(\Gamma, X, \mu)$ and let $\mu = \int_Z \mu_z d\eta(z)$ be the ergodic decomposition of b , so η -almost every measure μ_z is b -invariant and ergodic. Let b_z denote the action of Γ on (X, μ_z) , so that b_z is almost surely ergodic. Since b is free, η -almost every b_z is free. Hence, for η -almost every $z \in Z$ we have $b_z \prec a$. This implies $b \prec i_Z \times a$, where i_Z is the identity action of Γ on (Z, η) . Since Γ is finitely generated Corollary 10.14 gives $C(i_Z \times a) \leq C(b)$; since $C(i_Z \times a) = C(a)$, we have $C(a) \leq C(b)$. As $b \in \text{FR}(\Gamma, X, \mu)$ was arbitrary, this shows that $C(a) = C(\Gamma)$. Since $a \in A$ was arbitrary and A is dense G_δ in $(\text{FRERG}(\Gamma, X, \mu), w)$, this shows that $C^{**}(\Gamma) = C(\Gamma)$.

Page 79, lines 4-5: R. Tucker-Drob, Shift-minimal groups, fixed price 1, and the unique trace property, *arXiv: 1211.6395v3*, Corollary 6.22, shows that for infinite Γ and $a \prec b$, a free:

$$\begin{aligned} C(b) < \infty &\implies C(b) \leq C(a), \\ E_b \text{ treeable} &\implies C(b) \leq C(a). \end{aligned}$$

Page 87, line 18: Add “)” at the end of the formula.

Page 90, 12.11: Replace [HJ4] by [HJ5].

Page 90, Proof of 13.1: Here view (by picking representatives) the a_n, a as Borel actions and not equivalence classes of Borel actions up to equality a.e.

Page 114, line 9-: Replace “lemma” by “theorem”.

Page 123, line 7-: add “sofic” before Γ .

Page 123, (IIIb) and (IVb): G. Hjorth and A. Törnquist, The conjugacy relation on unitary representations, *Math. Res. Lett.*, **19(3)** (2012), 525–535, have shown that unitary equivalence for any Γ is Borel, in fact $\mathbf{\Pi}_3^0$.

Page 131, 3 lines before (B): generically \rightarrow generally.

Page 184, last lines of 30.4, 30.5: G -superrigid \rightarrow G -cocycle superrigid.

Page 185, third line of 30.6: G -superrigid \rightarrow G -cocycle superrigid.

Page 186, 10-: F_2 -superrigid \rightarrow F_2 -cocycle superrigid.

Page 186, 2-: G -superrigid \rightarrow G -cocycle superrigid.

Page 208, line 7 of (B): positive definite \rightarrow positive-definite.

Page 214, line 14: After “claim” add “applied to e_1, \dots, e_p and a given open set $W' \subseteq W$ ”.

Page 234: add to index
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