A.S. Kechris and B.D. Miller: Topics in Orbit Equivalence; Corrections and Updates (November 26, 2016)

Page VI, line 16: Add "n odd" in the parenthesis.

Page 3, lines 1,2: The assumption that the F_n are disjoint is not used in this proof.

Page 3, line 4: One can also replace "Since ... that" by "Since X is standard Borel,".

Page 27, Proposition 7.7: Christian Rosendal pointed out the following simpler proof of this proposition, which avoids the need for the Birkhoff ergodic theorem and the assumption that μ is invariant, by replacing the first half of the proof of Proposition 7.7 with the first half of the proof of Theorem 7.5.

Define $A_m, B_m \subseteq X$ exactly as in the proof of Theorem 7.5, and fix $m \in \mathbb{N}$ such that $\mu(B_m) + \mu(X \setminus A) < \epsilon$. Put $A'' = A_m$, and proceed as before: For each $x \in A''$, let $\ell''(x) > 0$ be the least natural number such that $T^{l''(x)}(x) \in A''$, set $k_0(x) = -n$, and recursively define $k_{i+1}(x)$ to be the least natural number such that $T^{k_{i+1}(x)}(x) \in A$ and $k_i(x)+n \leq k_{i+1}(x) \leq \ell''(x)-n$, if such a number exists. Define $B \subseteq X$ by

$$B = \{T^{k_i(x)}(x) : i > 0, x \in A'', \text{ and } k_i(x) \text{ is defined}\},$$

and note that $B \subseteq A$ and $\{T^i(B)\}_{i<n}$ is a pairwise disjoint family which covers $X \setminus (B_m \cup (X \setminus A))$, which is of measure $> 1 - \epsilon$.

Page 28, Remark 7.9: While this follows directly from Proposition 7.7 in the case that μ is invariant, it is false in general. Given $0 < \delta < \epsilon < 0.25$ and a natural number $n \geq 2$, there is an aperiodic Borel automorphism $T : X \to X$, a T-quasi-invariant probability measure μ on X, and a Borel set $A \subseteq X$ of measure $1-\delta$ which does not contain an (ϵ, n)-Rokhlin set of measure $\leq 1/n$. To see this, fix an aperiodic Borel automorphism $T' : X' \to X'$ which admits an invariant probability measure μ', set $X = \{(x, i) : x \in X' \text{ and } i < n\}$, define $T : X \to X$ by

$$T(x, i) = \begin{cases} (x, i+1) & \text{if } i < n-1, \\ (T'(x), 0) & \text{otherwise,} \end{cases}$$

and define μ on X by

$$\mu(B) = (1-\delta)\mu'(\mathrm{proj}_{X'}(B \cap X_0)) + \sum_{1 \leq i < n} \left(\frac{\delta}{n-1}\right) \mu'(\mathrm{proj}_{X'}(B \cap X_i)),$$

where $X_i = X' \times \{i\}$. Now suppose, towards a contradiction, that there is an (ϵ, n)-Rokhlin set $B \subseteq X \times \{0\}$ of measure $\leq 1/n$. Then

$$\mu(B) \leq 1/n \text{ and } \sum_{i<n} \mu(T^i(B)) > 1 - \epsilon.$$

It follows from the definition of μ that for $1 \leq i < n$,

$$\mu(T^i(B)) = \mu(B) \left(\frac{\delta}{n-1}\right) \left(\frac{1}{1-\delta}\right),$$

thus

$$\mu(B) \left(1 + \frac{\delta}{1-\delta}\right) > 1 - \epsilon.$$

It then follows that

$$\frac{1}{n(1-\delta)} > (1-\epsilon),$$

so $2 \leq n < 1/(1-\delta)(1-\epsilon)$, which is impossible, since $\delta < \epsilon < 0.25$.

It should be noted, however, that if we replace the requirement that $\mu(A) > 1 - \epsilon$ with the stronger hypothesis that

$$\mu\left(\bigcap_{i<n} T^{-i}(A)\right) > 1 - \epsilon,$$

then A does contain an (ϵ, n)-Rokhlin set of measure $\leq 1/n$. To see this, set

$$\delta = \epsilon - \mu\left(X \setminus \bigcap_{i<n} T^{-i}(A)\right),$$

appeal to Theorem 7.5 to find a (δ, n)-Rokhlin set $B' \subseteq X$ of measure $\leq 1/n$, and observe that the set $B = A \cap B'$ is as desired.

Page 45, line 6 : $\bigcup_{n\in\mathbb{N}} \bigcap_{m>n} \to \bigcap_{n\in\mathbb{N}} \bigcup_{m>n}$.

Page 45, line 5 of the proof of 10.5: Open parentheses after "$\forall x \in \mathrm{dom}(F_n)$".

Page 48, line 16-: Add after "identity)", "such that $f_n(x)Ex, \forall x \in S_n$".

Page 49, line 14-: A should also contain 1.

Page 50, line 4-: In the definition of f_n^α, α_n should be $\alpha(n)$.

Page 50, line 2 of Theorem 12.1: Add X after "space".

Page 52, line 6-: Replace C by $X_0 \cup \{x\colon (x,x) \in F_\infty^\alpha\}$; after "$F = E|X_0 \cup F_\infty^\alpha$" add: "to conclude that $\mu(A) = 0$ and thus, as A is a complete section of C, $\mu(C) = 0$."

Page 62, proof or 18.3: Julien Melleray pointed out that one can use the argument in the last paragraph of that proof to show that, for $\mu \in M_f$, we have that $C_\mu(E) < r$ holds iff

$$\exists\epsilon \in \mathbb{Q}^+\ \forall S \text{ finite} \subseteq \mathbb{N}\ \exists T \text{ finite} \subseteq \mathbb{N}\ [C_\mu(\Theta_T \sqcup \{\theta_i | D(\theta_i, \Theta_T)\}_{i\in S}) \leq r - \epsilon].$$

which directly shows that this condition is Borel on M_f.

Page 84, line 7: Replace "$x \in F$" by "xFy".

Page 89, line 17: After "where" add "$\bar{A}^0_\theta = A_\theta$ and".

Page 100, line 7: The first $A^n_{i'}$ should be $A^{n+1}_{i'}$.

Page 100, line 11: A^n_{i+1} should be A^{n+1}_i.

Page 102, lines 4 and 5: The exponent of φ_∞ should be n_0 in both cases, not n.

Page 102, line 24: The second $\pi_{n,m}$ should be $\pi_{n,m}(\theta)$.

Page 102, line 14-: $\{\varphi_k)_{k\in K}$ should be $\{\varphi_k\}_{k\in K}$.

Page 102, line 2-: ψ_i should be $\tilde{\psi}_i$.

Page 103, line 3: "extend $\tilde{\varphi}_0$" should be "extend $\tilde{\varphi}$".

Page 106, line 6-: Replace "$E|S_e$" by "$F_e|S_e$".

Page 107, last sentence of Section 28: Replace this sentence by: "Note again that the argument in the proof of 28.8 shows that in 28.14 the following weaker statement is true: either (I)* $C_\mu(E) = 1$ or (II) holds."

Page 108, line 2-: 18.5 should be 18.6.

Page 110, line 2-: $(g \cdot x, h \cdot x)$ should be $(g^{-1} \cdot x, h^{-1} \cdot x)$.

Page 115: Damien Gaboriau has pointed out still another way of seeing that the cost of any infinite amenable group is 1. Suppose, towards a contradiction, that such a group Γ acts freely on a standard Borel space X in a Borel way with invariant ergodic probability measure μ, and $C_\mu(E_\Gamma^X) > 1$. By Lemma 28.12, there is a Borel subtreeing $\mathcal{T} \subseteq E_\Gamma^X$ generating an ergodic equivalence relation $E_\mathcal{T}$ of cost strictly greater than 1. Since subequivalence relations of μ-amenable equivalence relations are μ-amenable, it follows that $E_\mathcal{T}$ is μ-amenable. So from [JKL, 3.23] (which generalizes a result in [A1]), we have that almost every component of $\mathcal{T}$ has at most 2 ends, from which it follows (see, e.g., [JKL, 3.19]) that $E_\mathcal{T}$ is hyperfinite a.e., so has cost 1, a contradiction.

Page 115: After 31.1 add:

Part i) follows from 9.2 and 10.2 and a proof of part ii) is essentially contained in Example 9.4.

Page 121, 35.5: Ioana has extended this result by weakening normality to almost normality and dropping the assumption that N has fixed price.

Pages 123 and 128: Problem 35.7 has been solved by Abert and Nikolov. The answer is negative. See: M. Abert and N. Nikolov, Rank gradient, cost of groups and the rank versus Heegard genus problem, arXiv:math/0701361v3.