1. Show that if \( \phi_1 : U_1 \rightarrow S \) and \( \phi_2 : U_2 \rightarrow S \) are diffeomorphisms onto a domain \( U \subset S \), then the area of \( \phi_1 \) and the area of \( \phi_2 \) are equal.

2. Let \( \phi : S^2 \rightarrow \mathbb{R}^3 \) be the map given by \( \phi : (x, y, z) \mapsto (3x, 2y, z) \). Compute the first fundamental form and the area of \( \phi \).

3. Let \( \gamma : (-3\pi, \frac{\pi}{3}) \rightarrow S^2 \) be the map given by
   \[
   \gamma : t \mapsto \left( \frac{t}{3\pi} \cos t, \frac{t}{3\pi} \sin t, \sqrt{1 - \frac{t^2}{9\pi^2}} \right).
   \]
   Compute the length of \( \gamma \).

4. Show that every smooth surface has a non-zero vector field.

5. Let \( S \) be a smooth surface and \( \phi : S \rightarrow \mathbb{R}^3 \) be a smooth immersion. Show that \( S \) is orientable if and only if there is a smooth vector field \( X \) along \( \phi(S) \) so that
   \[
   \begin{align*}
   &\bullet \ X(p) \neq 0 \text{ for all } p \in S. \\
   &\bullet \ \langle X(p), V \rangle_{\phi(p)} = 0 \text{ for all } V \in \phi_*p(T_pS).
   \end{align*}
   \]
   Here, \( \langle \cdot, \cdot \rangle_{\phi(p)} \) is the standard inner product on \( T_{\phi(p)}\mathbb{R}^3 \).
   Hint: Use partitions of unity.

6. Use Problem 5 to show that \( S^2 \) is orientable, and that the Möbius strip \( ((0, 1) \times [0, 1])/(y, 0) \sim (1 - y, 1) \) is not orientable.

7. Show that there are only two (equivalence classes of) orientations on any connected orientable smooth surface.

8. Let \( \phi : S_1 \rightarrow S_2 \) be a diffeomorphism.
   (a) Prove that given an orientation on \( S_1 \), \( \phi \) induces an orientation on \( S_2 \).
   (b) Suppose \( S_1 = S_2 = S \). Is the orientation induced by \( \phi \) and the orientation we started with necessarily equivalent? If so, prove it, and if not, give an example.