Due: Friday, February 17, 2017 at noon.

All numbered problems are from Dummit and Foote, Third Ed.
All problems will be graded. Show all work to receive full credit.


• Problems 5, 8 in sec 10.1;
• Problems 8, 13 in sec 10.2.
• Problems 5, 18 in sec. 10.3.

Let $M$ be a cyclic module, $m \in M$ a generator of $M$. For any left ideal $I$ of $R$ and any $R$-module $N$, let

$$N[I] = \{ n \in N | an = 0 \forall a \in I \}.$$

Prove that
(1) $N[I]$ is closed under addition;
(2) for all $R$-module $N$: $\text{Hom}_R(M, N) \cong N[\text{Ann}(m)]$, as abelian groups;
(3) (Schur’s lemma) If $M$ is irreducible, then $\text{End}_R(M)$ is a division ring.

Assume now that $R$ is commutative. Maintaining notations and assumptions as above, prove that
(1) $N[I]$ is a sub-$R$-module of $N$;
(2) $\text{Hom}_R(M, N)$ and $N[\text{Ann}(m)]$ are isomorphic as $R$-modules;
(3) If $M$ is irreducible, then $\text{End}_R(M)$ is a division ring containing in its center the field $F = R/\mathfrak{M}$, for $\mathfrak{M} = \text{Ann}(m)$.

The following problem is for extra credit:

• Let $R, S$ be two rings with units. Prove that
  (1) An $R$-module $M$ can be made into a $R \times S$-module via
      $$(r, s) \circ m = rm$$
      for all $m \in M$.
  Similarly, an $S$-module $N$ can be also made into a $R \times S$-module, via
      $$(r, s) \circ n = sn$$
      for all $n \in N$.
  (2) Every $R \times S$-module $T$ splits canonically as $T = M \oplus N$ for some modules $M, N$ as above.
      (Hint: problem 15 from section 10.3)