Due: Friday, January 27, 2017 at noon.

All numbered problems are from Dummit and Foote, Third Ed.
All problems will be graded. Show all work to receive full credit.

Read sections: 7.6, 8.1, 8.2 and 8.3 of the textbook.

- From section 7.6: problem 3.

  - Let $F$ be a field and $F[x]$ the rings of polynomials in one variable $x$ and coefficients in $F$. Let $f(x) \in F[x]$ of degree $n$ and assume $f(x)$ is a product of distinct linear factors, i.e.

  $$f(x) = \prod_{i=1}^{n} (x - a_i)$$

  where $a_i \in F$ are all distinct.

  Prove that the ring $F[x]/(f(x))$ is isomorphic to $F^n$.

- From section 8.1: problems 4 and 11 but under the assumption that $R$ is a PID (not an ED).

- From section 8.2: 4 and 7.

  \textit{(Hint to problem 4):} Rephrase part (ii) in terms of ideals in $R$. Use Zorn’s Lemma.

The following problem is for extra credits:

- Let $F$ be a field, write $R = F[[X]]$ for the ring of formal power series in the indeterminate $X$ with coefficients in $F$ (see problem 3 of section 7.2 for definition), and $Q = F((X))$ for the ring of formal Laurent series (see problem 4 of section 7.2 for definition). Prove that

  1. $R$ is an integral domain.
  2. A power series $f(x) \in F[[x]]$ is a unit if and only if $f(0) \neq 0$ (cfr. with problem 3 part (c)).
  3. $Q$ is a field (i.e. problem 4 part (a)).
  4. $Q$ is the fraction field of $R$. 