Caltech Department of Mathematics

Ma 3/103 KC Border
Introduction to Probability and Statistics Winter 2016

Assignment 5: Stochastic Processes

Due Tuesday, February 14 by 4:00 p.m. at 253 Sloan

Instructions:

When asked for a probability or an expectation, give both a formula and an explanation for why you used that formula, and also give a numerical value when available.

When asked to plot something, use informative labels (even if handwritten), so the TA knows what you are plotting, attach a copy of the plot, and, if appropriate, the commands that produced it.

No collaboration is allowed on optional exercises.

You may need to refer to the notes for Lecture 15.

Exercise 1 (Queueing)

Every day, a number of automobiles arrive in the morning at Casey's Auto Body Shop. Casey has room for one car in the shop and up to two on the lot. If there is no room for a car, Casey is forced to send it to his competitor, Bob's Body Boutique. Once a car has been dropped off at Casey's it remains there until it is repaired. It takes a whole day to repair a car, but once repaired, it is gone by the following morning.

Assume the number of autos that arrive each morning follows a Poisson(μ) distribution, and that the numbers on different days are independent.

The situation at Casey's can be viewed as a five-state Markov chain, where the states are 0, 1, 2, 3, B, where the numeric states refer to the total number of cars at Casey's, and the state B is that there are 3 cars at Casey's *and* he had to send some overflow to Bob.

- (10 pts) Draw the graph corresponding to this Markov chain.
- (10 pts) Write down the transition matrix for this in terms of the Poisson pmf $p_\mu(k)$.
- (10 pts) For $\mu = 0.2$ and $\mu = 1.5$, write down the numeric transition matrix.
- (10 pts) Today, Casey has no cars. For $\mu = 0.2$ and $\mu = 1.5$, what is the numeric probability of each state in 30 days.
- (10 pts) For $\mu = 0.2$ and $\mu = 1.5$, numerically find the steady-state (invariant) distribution of the Markov chain.
- (10 pts) For $\mu = 0.2$ and $\mu = 1.5$, what is the steady-state probability that Casey will send Bob some customers on April 1. (Hint: It's the probability of state B under the invariant distribution.)

□

Exercise 2 (10 pts) How much time did you spend on the previous exercises? □

Exercise 3 (Optional Exercise) (30 pts) Consider the experiment of tossing a coin until you get two consecutive heads. What is the probability distribution of the number of tosses?

(Just to be clear, the sequence HH requires two tosses and $HTTHH$ requires five tosses.) □