

Assignment 1: Utterly trivial probability calculations

Due Monday, January 9 by 4:00 p.m at 253 Sloan

Instructions: When asked for a probability, give both a formula and an explanation for why you used that formula, and also give a numerical value when available. When asked for a numerical probability, evaluate the formula numerically.

When a problem says that an element of a set is selected **at random**, assume that each element is equally likely to be chosen.

Because of the unusually short first week of this term, there is not much I can ask you about. I thought about skipping this week, but here are some really easy questions that do have a point to them. I mean really easy—you only need to be able to count to one hundred (actually, not that high). Oh, and you have to divide. I hope you are not too bored. (They are inspired by a textbook intended for MBA students.) Next week will be more challenging.

Exercise 1 (10 pts) A classic trope in probability is repeatedly tossing a coin. Here is an example.

A fair coin is tossed three times.

- (4 pts) Make a list of all the outcomes in the sample space for this experiment. How many points does it contain?
- (3 pts) What is the numerical probability that there are two consecutive Tails?
- (3 pts) What is the probability that all three tosses yield distinct outcomes?

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Exercise 2 (15 pts) Another classic trope in probability is selecting an object at random from an urn. Here is an example.

An urn contains five balls numbered 1 through 5. Two balls are drawn at random without replacement. Let B_1 be the number on the first ball drawn and B_2 be number on the second one drawn. These are random variables.

- (3 pts) Make a table of all the outcomes in the sample space for this experiment. How many points does it contain?
- (3 pts) What is the numerical probability that the largest number drawn will be 3?
- (3 pts) What is the numerical probability that the largest number drawn will be 4 or more?
- (3 pts) What is the numerical probability that the sum of the numbers drawn will be even?
- (3 pts) What is the numerical probability that the sum of the numbers drawn will be a multiple of three?

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Exercise 3 (15 pts)

A tiny day-care center has six children from three different families. One family has one child in the center, another has two, and the third family has three. All three families have all their children in the center.

What is the probability that a randomly selected child is the first-born of its family? The answer depends on the sampling scheme. Consider the two following schemes:

1. There is an urn for each family, which contains all the children in that family. A family is selected at random, and then a child is selected randomly from the family urn.
2. All the children are put into one urn, and a child is selected at random.

What is the probability under each of these two sampling schemes? (Later on, you may be asked to generalize this example.)

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Exercise 4 (5 pts) How much time did you spend on the previous exercises?

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