Caltech Department of Mathematics

Ma 3/103
Introduction to Probability and Statistics

KC Border
Winter 2017

Assignment 0: Coin flipping

Due: Thursday, January 4, 8:00 p.m., on line.

Coin flipping is the canonical example of a random experiment. A number of authors have written on why this completely deterministic physical process can appear to be random. An understandable example is a paper by Karl Menger [2] who argues that in the space of initial conditions (height and rotational velocity) approximately equal areas give rise to Heads and to Tails. More recently, Persi Diaconis, Susan Holmes, and Richard Montgomery [1], using a more elaborate physical model and high-speed photography, argue that a coin that is flipped and caught should turn up the same way as it started about 51% of the time.

During the first lecture I will distribute dollar coins, obtained from the Caltech Employees Federal Credit Union. **It ask that you use one of these coins, to make the coins as uniform as possible.** Your first assignment is to flip the coin 128 ($= 2^7$) times and record the sequence of results (Heads or Tails), using the protocol described below. ("Heads" is the side of the coin that shows someone's head, often a president, or Susan B Anthony or Sacajawea, but not the Statue of Liberty. "Tails" is something else, often the Statue of Liberty. If in doubt, ask around for the consensus opinion.)

If you miss the first lecture, you may obtain an official coin from Barbara Estrada in 111 Baxter during normal business hours, starting after 1:00 p.m. on January 4. Keep your coin until the end of the term, as you may need it again. At the end of the term I would like the equivalent cash value returned to me. You may keep your coin if it brings you luck. (The coins are provided by me personally, not the Math Department.)

This is not an idle exercise. The data from these sequences will be used for a number of purposes. First, I hope to illustrate that the Laws of Chance enable us to predict with reasonable accuracy such things as the number of Heads and the number of runs of length 1, 2, or 3. We will also use the results to generate approximately uniform random variables that can then be used to generate any other kind of random variable. When we start on statistics, we will use these data to test the Diaconis–Holmes–Montgomery

hypothesis. The number of HW points is yet to be determined, but will be about 10% of the HW total.

In order to save time and minimize transcription errors, I have created a form to submit your results at http://www.math.caltech.edu/%7E2016-17/2term/ma003/CoinTossResults.html.

Experimental Protocol

1. Work in groups. Assign half of the group a starting position of Heads and the other half Tails. (If there is an odd number in your group, flip a coin to determine the starting position of the odd person out.) Half the group can record while the other half flips. Then switch and let the recorders become flippers, and vice-versa. **Everyone should flip their own coin and report their own 128 tosses.** You might find that imbibing a beverage such as coffee will make the task more enjoyable and elevate the conversation. If you record the results on a computer, you can just cut and paste the results.

2. Please encode the results as sequence of 0s and 1s, with 0 = Heads, and 1 = Tails. Do not include any white space.

3. Diaconis–Holmes–Montgomery are not explicit about the exact protocol for flipping a coin, but based on [1, § 5.1 and § 6.5] here is my version: Make a fist with your thumb tucked slightly inside. Position the coin on top of your thumb-fist with Heads or Tails facing up, depending on your assigned starting position. Give the coin a "normal flip" with your right hand if you are right-handed (left hand if you are left-handed), catching it in the same hand (without bouncing) at approximately the same height. Try to flip it high enough and/or with enough spin so it flips several times.

   Open you hand palm up, and look at which side of the coin is facing up and record it. Do *not* flip the coin over after catching it to put it on you arm, as many people do. (Or if you do flip it over, report the reverse side.)

   Don't worry too much about your flipping and catching technique. This is not a test of your coordination or dexterity. If you miss catching the coin, pretend it never happened.

4. Record the results of 128 flips in the order that they occur. Do not sort them!

5. Double check to make sure you report 128 tosses. Every year there are a handful of students who submit 127 or fewer tosses. This is annoying and creates extra work for me when I have to contact you to do it over.

6. Submit the results via the form at http://www.math.caltech.edu/%7E2016-17/2term/ma003/CoinTossResults.html. If there is anything you do not understand, ask me or your TA for clarification.

References

[1] P. Diaconis, S. Holmes, and R. Montgomery. 2007. Dynamical bias in the coin toss. *SIAM Review* 49(2):211–235. DOI: 10.1137/S0036144504446436

[2] K. Menger. 1954. Tossing a coin. *American Mathematical Monthly* 61(9):634–636. http://www.jstor.org/stable/2307682