Caltech Department of Mathematics

Ma 3/103 KC Border
Introduction to Probability and Statistics Winter 2017

Midquarter Examination

Due Wednesday, February 8, at 4:00 p.m.
in the Exam Drop Box outside 253 Sloan.

Instructions

This exam has seven questions. Many of them are short. You have **4.5 hours in one sitting** to answer them.

- Please use a **Caltech blue book** and put your name and section on the front cover.
- There is no collaboration allowed.
- You may use the textbooks (by Pitman and by Larsen and Marx), homework solutions, lecture notes, and other handouts from the current Ma 3 web site, your own notes and homework, and TA notes. You may also use someone else's notes that you have copied by hand.
- You may not use any internet sources other than the current course web page.
- You may use calculators and computers to do numerical calculations, and to use elementary built-in functions like logs and normal distribution functions (as an alternative to using tables).
- Please indicate clearly any work done in overtime. Points will be recorded separately and considered informally in course grades.
- Write legibly in complete sentences and explain yourself.
- When asked for a numerical probability be sure to explain how you calculated it.
- If you have any questions about these instructions, consult a TA for the course (they are on the course web page), the lead TA William Chan <wcchan@caltech.edu>, or the professor <kcb@caltech.edu>.

1. (20 pts) Two points are picked on the unit circle by choosing angles at random from a uniform distribution on $[-\pi, \pi]$.

   - What is the probability that they lie in the same quarter-circle? Explain your reasoning.

2. (30 pts) Let $X_1, \dots, X_n$ be independently and identically distributed with mean μ and variance σ^2. Let $\bar{X} = (X_1 + \cdots + X_n)/n$.

   - Calculate the covariance and the correlation between X_1 and $\bar{X}$.

3. (20 pts) There are three urns holding various numbers of black, white, and yellow balls. Here is a table showing their composition:

Urn	B	W	Y
1	10	15	5
2	30	6	4
3	7	7	2

   An urn is selected at random, and a ball is drawn out.

   - For each $i = 1, 2, 3$, what is the conditional probability that urn i was selected, given that a yellow ball was drawn?

4. (30 pts) An urn contains 20 Blue balls and 10 Green balls. A sample of two balls is drawn at random *with replacement*. You are informed that at least one sample ball is Green. Let B denote the number of Blue balls in the sample and let G denoted the number of Green balls.

   - Compute the expected values of B, G, and B/G, conditional on there being at least one Green ball.

5. (30 pts) An urn contains 10 Blue balls and 5 Green balls. Balls are drawn at random *without replacement* until a Blue ball is drawn.

   - For each $k = 1, \dots, 15$, what is the probability that drawing stops with the k^{th} draw?
   - What is the expected number of Green balls drawn? What is the expected number of Blue balls drawn?

6. (30 pts) Let X and Y be independent Uniform[0,1] random variables. (The density is $f(t) = 1$ for $0 \leqslant t \leqslant 1$, and $f(t) = 0$ otherwise.)

   - Find the density for $X + Y$. (Explain your reasoning.)

7. (30 pts) According to the National Collegiate Athletic Association's National Center for Catastrophic Sport Injury Research,[1] for the years 1992–2012 (inclusive) there were 23 instances of athletes, coaches, officials, or spectators being struck by either a discus, shot (as in shot put), or javelin at high school and college track meets or practices in the U.S. Of these injuries, 4 were fatal.

   - Based on these facts, choose and justify a probability model for the number of such strikes annually, and use it to answer the following questions.
   - What is the probability that there would be no strikes in 2017?
   - In 2002, three athletes were struck by a shot put and one by a discus; a coach was struck by a shot put. What is the probability that there would be five strikes in 2017?

[1] Source: `http://nccsir.unc.edu/files/2014/06/NCCSIR-30th-Annual-All-Sport-Report-1982_2012.pdf`, Table 1, pp. 25–26.