In the sequel, \( V \) denotes a vector space defined over the field \( \mathbb{F} = \mathbb{R} \) or \( \mathbb{C} \) unless otherwise specified.

1. Read from the textbook: Chapter 5, Section 4-6, Chapter 6, Section 1-2, Chapter 7, Section 1, 3-5.

2. [20pts] From the textbook: Chapter 5, Problem 4.3.

3. [20pts] From the textbook: Chapter 5, Problem 6.1 (only second and third matrices).

4. [40pts] Let \( v_1 = (1, 0, 1)^T, v_2 = (2, 2, 0)^T \), and \( V \) be the subspace in \( \mathbb{R}^3 \) spanned by \( v_1 \) and \( v_2 \).
   
   (a) Find \( \{u_1, u_2\} \) an orthogonal basis of \( V \).
   
   (b) For \( i = 1, 2 \), express \( v_i \) as a linear combination of the new basis.
   
   (c) Compute the orthogonal complement of \( V \) in \( \mathbb{R}^3 \).
   
   (d) Complete \( \{u_1, u_2\} \) to an orthogonal basis of \( \mathbb{R}^3 \).
   
   (e) Let \( w_1 = (1, -1, -1)^T, w_2 = (1, 1, 1)^T \). For each of \( w_i \), determine whether \( w_i \) belong to the space \( V \). If possible, write \( w_i \) as a linear combination of \( v_1, v_2 \). If not, find the distance from \( w_i \) to \( V \).
   
   (f) For \( w = (3023, 2345, 678)^T \): does \( w \) belong to the space \( V \)? (Hint: do not compute the distance!)

5. [20pts] Let \( T : V \to V \) be a self-adjoint linear operator on a real vector space \( V \). Assume that \( \langle Tv, v \rangle \geq 0 \) for every \( v \in V \). Show that for every positive integer \( k \), there is a linear operator \( S : V \to V \) such that \( T = S^k \).