This is a one hour timed quiz. You may refer to the course textbook or to your course notes, but no other sources (including humans).

Do not look at the next page until you are ready to start!
Define a sequence of integers \( g_0, g_1, g_2, \ldots \) by the formulas \( g_0 = 0 \), \( g_1 = 1 \), and for \( n \geq 2 \), \( g_n = g_{n-1} + 2g_{n-2} \). (Just like the Fibonacci sequence except for the factor of 2.) Consider the following matrix:

\[
A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}.
\]

(1) If we set \( v_n = \begin{pmatrix} g_n \\ g_{n+1} \end{pmatrix} \), show that \( A \cdot v_n = v_{n+1} \).

(2) Deduce that \( A^n \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = v_n \).

(3) Find a matrix \( C \) such that \( C^{-1} AC = D \) is diagonal.

(4) Use the fact that \( A^n = CD^nC^{-1} \) to find a closed formula for \( A^n \) and hence for \( g_n \).