This is a ninety minute timed quiz. You may refer to the course textbook or to your course notes, but no other sources (including humans).

Do not look at the next page until you are ready to start!
(1) Let $U$ and $W$ be the following subspaces of $\mathbb{R}^4$:

$U = \{(x, y, z, w) \in \mathbb{R}^4 | x + y + z + w = 0 \}$

$W = \{(x, y, z, w) \in \mathbb{R}^4 | x - y = 0 = z - w \}$.

(a) Compute the dimensions of $U \cap W$ and $U + W$.

(b) Pick bases for $U$, $W$, and $U + W$ such that the basis of $U + W$ contains the bases of $U$ and $W$.

(2) Let $T : V = \mathbb{R}^3 \to W = \mathbb{R}^3$ be the linear transformation

$T(x, y, z) = (x + y + z, y + z, z)$

(a) Let $E$ denote the standard basis of $\mathbb{R}^3$. Prove that there exists a basis $B$ of $\mathbb{R}^3$ such that matrix representing $T$ with respect to the bases $B$ of $V$ and $E$ of $W$ is the identity matrix (without finding a basis).

(b) Find such a basis $B$. Is $B$ unique?

(c) Compute the matrix representing $T$ with respect to the bases $E$ of $V$ and $B$ of $W$. 