Math 1b Analytical – Homework Set 9
Due 4:00 pm on Monday, March 6

Read from the textbook: Chapter 1, Sections 11-16 and Chapter 5.

(1) From Apostol, chapter 1.13: Problem 11.
   Also, compute an orthonormal basis for the subspace consisting of polynomials of degree at most 3.

(2) Let $V = \mathbb{R}^4$, and $U = L(v_1, v_2)$ be the subspace of $V$ generated by the vectors $v_1 = (1, 3, 1, 1)$ and $v_2 = (3, 2, 2, 1)$.
   • Find an orthogonal basis of $U$.
   • Find the orthogonal projection of $v = (1, 1, 1, 1)$ onto $U$.
   • Find the distance of $v = (1, 1, 0, 0)$ from $U$.

(3) Let $L_{a,b}$ in $\mathbb{R}^2$ be the straight line defined by the equation $y = a + bx$ for some $a, b \in \mathbb{R}$. Given a collection of points $S = \{(x_i, y_i)\}_{i=1,\ldots,k}$ in $\mathbb{R}^2$: a line $L$ is called the least squares fit to $S$ if it minimizes the total quadratic error $\sum_{i=1}^{k} |a + bx_i - y_i|^2$. (E.g., this would be the line passing thru all points of $S$ if it existed.) Find the line that best fits the points $(-2, 4), (-1, 3), (0, 1), (2, 0)$ in this sense.

(4) Let $v_1 = (1, 0, 1, 0), v_2 = (2, 2, 0, 0)$, and $V = L(v_1, v_2)$ in $\mathbb{R}^4$.
   (a) Find $\{u_1, u_2\}$ an orthogonal basis of $V$.
   (b) For $i = 1, 2$: express $v_i$ as a linear combination of the new basis.
   (c) Compute the orthogonal complement of $V$.
   (d) Complete $\{u_1, u_2\}$ to an orthogonal basis of $\mathbb{R}^4$.
   (e) For $w = (1, -1, -1, 0)$: does $w$ belong to the space $V$? If possible, write $w$ as a linear combination of $u_1, u_2$.
   (f) For $w = (1, 1, 1, 1)$: does $w$ belong to the space $V$? If not, find the distance from $w$ to $V$. 