(1) From Ch. 4.10 in the textbook: Problems 4, 5, 7, and 8.

(2) A population of fish is divided into red, blue and yellow. It has been observed that 1/4 of the descendants of red fish are red, 1/2 are blue and 1/4 yellow; descendants of blue fish are 1/3 red, 1/3 blue and 1/3 yellow; descendant of yellow fish are 1/3 red, 1/9 blue and 5/9 yellow. We record these data in the following probability matrix

\[
P = \begin{pmatrix}
\frac{1}{4} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{2} & \frac{1}{3} & \frac{1}{9} \\
\frac{1}{4} & \frac{1}{3} & \frac{5}{9}
\end{pmatrix}
\]

Assume the current population \( u_0 \) is 1/3 red, 1/3 blue and 1/3 yellow.

(a) Compute (if it exists!) a population \( v_1 \) which remains unchanged (stable) as generations pass.

(b) Compute all eigenvalues and eigenvectors of the matrix \( P \).

(c) Compute a closed formula for the \( n \)-th generation \( u_n \).

(d) Prove that indeed with the passing of generations the fish population approaches \( v_1 \) (i.e. \( \lim_{n \to \infty} u_n = v_1 \)).