Math 1b Analytical – Homework Set 6
Due 4:00 pm on Monday, February 13, 2015

Read from the textbook: Chapter 3, Sections 1–17.

You can collaborate on the problems as long as you write up all solutions in your own words and understand those solutions.

(1) From Ch. 3.11 in Apostol: Problem 1
(2) From Ch. 3.17 in Apostol: Problem 5(a)
(3) Let $M$ be a $n \times n$-matrix, written in blocks form:

\[
M = \begin{pmatrix}
A & D & E \\
0 & B & 0 \\
0 & F & C \\
\end{pmatrix}
\]

where $A, B, C$ are square matrices. Prove that $\det M = \det(A) \det(B) \det(C)$.

(4) Let $A$ be a $3 \times 3$ matrix

\[
A = \begin{pmatrix}
1 & 0 & 1 \\
\alpha & 1 & 0 \\
2 & \alpha & 1 \\
\end{pmatrix}
\]

for which value of $\alpha$ is $A$ singular (a square matrix $A$ is called singular if it is not invertible)?

(5) (a) Let $V$ be a vector space and $W$ be a subspace of $V$. Define $\text{Ann}(W) \subset \mathcal{L}(V, F)$ to be the set of all linear maps $\phi : V \to F$ such that $\phi(w) = 0$ for all $w$ in $W$. Show that if $V$ is $n$ dimensional and $W$ is $r$ dimensional then $\text{Ann}(W)$ is $n - r$ dimensional.

(b) Let $A$ be an $m \times n$ matrix, and let $T$ be the associated linear transformation from $F^n$ to $F^m$. We have seen that by identifying $F^n$ and $F^m$ with the space of $n \times 1$ matrices and $m \times 1$ matrices, that we can think of $T$ as being given by the map $v \mapsto A \cdot v$ where the dot is matrix multiplication. In this way, we can identify the null space of $T$ with the set of $n \times 1$ matrices $\{v \in M_{n,1}|A \cdot v = 0\}$. Consider instead the subset of the space of $1 \times m$ matrices $\{C \in M_{1,m}|C \cdot A = 0\}$. Show that we can identify this space with $\text{Ann}(T(F^n))$.

(c) Use the first two parts of this exercise, together with the rank nullity theorem, to show that the dimension of the subspace of $M_{n,1}$ spanned by the columns of $A$ is equal to the dimension of the subspace of $M_{1,m}$ spanned by the rows of $A$. 

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