1) a) Compute the Zeta function

$$\zeta_K(s) := \prod_v \frac{1}{1 - N_v^{-s}}$$

of the field $K = \mathbb{F}_q(t)$. Here the Euler product is over all discrete valuations $v$ of $K$ and $N_v$ denotes the cardinality of the residue field of $v$.

b) Show that the number $i(n)$ of monic irreducible polynomials of degree $n$ with coefficients in $\mathbb{F}_q$ is

$$i(n) = \frac{1}{n} \sum_{d|n} \mu(d)q^{n/d}$$

where $\mu$ is the Moebius function (you might have to read up on the Moebius inversion formula). Prove that $i(n) > 1$ for any $n \geq 1$.

2) Do Ch. 7 exercise 41 (you may freely use results of previous exercises and theorems with proper quotation).