1. A function is *monotonically increasing* if for every $i > j$ we have $f(i) \geq f(j)$. How many monotonically increasing functions are there from $\{0, 1, 2, 3, \ldots, n\}$ to $\{0, 1, 2, 3, \ldots, n\}$? (hint: Define $k_i = f(i) - f(i-1)$, $k_0 = f(0)$, and $k_{n+1} = n - f(n)$. What is the meaning of these $k_i$’s?).

2. Let $G$ be a permutation group of a set $X$, such that $|X| - 2 \geq |G| \geq 2$ and that there are exactly two distinct orbits. Prove that there exists a permutation $g \in G$ that does not contain any cycles of length one in its cycle structure.

3. (Probably too difficult to actually be on the exam) Biscuit decides to make a Hollywood movie. He of course receives offers from $m$ different producers, where the $i$’th producer suggests to give a funding of $X_i$. Biscuit can accept funding from as many producers as he likes. However, every producer is willing to invest in the movie only if each of her favorite actors is in it (every producer has a list of such actors, and the lists may have common actors in them). In total, the lists contain the names of $n$ distinct actors, and the $i$’th actor requires a fee of $Y_i$. Since Biscuit is only doing this for the money, he would like to maximize the funding gained minus the salaries paid. Describe an efficient algorithm that can help Biscuit (hint: build a flow network and consider only the cuts of this network. The flows are not important.)