The following questions are taken from last year\'s first assignment.

1. In class we used induction to prove the prime decomposition property, and we proved by contradiction that there are infinitely many primes. Use contradiction to prove the prime decomposition property, and induction to prove that there are infinitely many primes.

2. Given $a, b \in \mathbb{N}$, the least common multiple of $a$ and $b$, also written as $LCM(a, b)$, is the smallest natural number $c \in \mathbb{N}$ such that $a|c$ and $b|c$. For example, $LCM(5, 25) = 25$ and $LCM(6, 21) = 42$. Prove that for any pair $a, b \in \mathbb{N} \setminus \{0\}$, we have

$$LCM(a, b) \cdot GCD(a, b) = a \cdot b$$

(hint: write $a$ and $b$ in the form $p_1^{a_1}p_2^{a_2} \ldots p_k^{a_k}$ where $p_1, \ldots, p_k$ are distinct primes).