Ma 2- Written Homework #4
Due Monday, October 24, 2016 before 4pm

Name (Print):

Please write down the question number at the beginning of your solution. You can use this sheet as a cover.

1. (10 points) Section 3.2
If the differential equation \( ty'' + 2y' + te^t y = 0 \) has \( y_1 \) and \( y_2 \) as a fundamental set of solutions, and if \( W(y_1, y_2)(1) = 5 \), then find the value \( W(y_1, y_2)(2) \).

Please justify your answer.

2. (10 points) Section 3.2
Show that if \( p \) is differentiable and \( p(t) > 0 \), then the Wronskian \( W(t) \) of two solutions of \( [p(t)y']'' + q(t)y = 0 \) is \( W(t) = c \frac{e}{p(t)} \), where \( c \) is a constant.

Please justify your answer.

3. (10 points) Section 3.3
An equation of the form
\[
 t^2 \frac{d^2 y}{dt^2} + \alpha \frac{dy}{dt} + \beta y = 0, \; t > 0, 
\]  
(1)

where \( \alpha \) and \( \beta \) are real constants, is called Euler equation.

(1) Let \( x = \ln t \), then show that the equation is transformed into the form
\[
 \frac{d^2 y}{dx^2} + (\alpha - 1) \frac{dy}{dx} + \beta y = 0. 
\]  
(2)

(2) Solve the equation for \( \alpha = -1 \), and \( \beta = 5 \).

Please include the detailed steps in answers.

4. (10 points) Section 3.4
Solve the initial value problem
\[
 4y'' + 12y' + 9y = 0, \; y(0) = 1, \; y'(0) = -4. 
\]  
(3)
5. (10 points) Section 3.4
Use the method of reduction of order to find a second solution to the given differential equation.

\[(x - 1)y'' - xy' + y = 0, \ x > 1, \ y_1 = e^x.\]

Please justify your answer.

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<thead>
<tr>
<th>Question:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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