(1) For $a \in A$ have $-a \in -A \Rightarrow -a \leq \sup(-A)$. \\
Therefore, for any lower bound $y \in \mathbb{R}$ of $A$ it's clear that $y$ is an upper bound of $-A$, so \\
$\sup(-A) \leq -y \Rightarrow -\sup(-A) \geq y$ \\
and therefore $-\sup(-A) \geq \inf(A)$.

(2) For $x \in A+B$ write $x = a+b$ with $a \in A$, $b \in B$. Then \\
$x = a+b \leq \sup A + \sup B$ \\
so that $\sup(A+B) \leq \sup A + \sup B$. Conversely, for every upper bound $y$ of $A+B$ one has \\
y $\geq a+b$ \\
$\Rightarrow y-a \geq b$ \\
$\Rightarrow y-a \geq \sup B$ \\
$\Rightarrow y-\sup B \geq a$ \\
$\Rightarrow y-\sup B \geq \sup A$ \\
$\Rightarrow y \geq \sup A + \sup B$ \\
and therefore $\sup(A+B) \geq \sup A + \sup B$. 
No. 2
Since $n$ is even one has
\[
\lim_{x \to -\infty} f(x) = \infty \quad \text{and} \quad \lim_{x \to +\infty} f(x) = \infty.
\]
So, there is an $R > 0$ s.t.
\[
|\x| > R \implies |f(x)| > f(0).
\]
$f([-R,R])$ attains a global minimum at some $c \in [-R,R]$ by extremum value theorem. But then $c$ must be a global minimum for $f$ on all of $\mathbb{R}$ since
\[
f(c) \leq f(0) < f(x)
\]
for all $x \in \mathbb{R} \setminus [-R,R]$. 
No. 3
Let \( f(x) = \sqrt{x} \) and \( a = 0 \), \( L = f(0) = 0 \). Then for \( \delta = \varepsilon = 1 \) we have
\[
|f(x)| < \varepsilon.
\]
However, \( |x| < \frac{1}{2} \) \( \Rightarrow \) \( |f(x)| = |\sqrt{x}| < \frac{1}{2} \) is NOT true since e.g. for \( x = \frac{1}{2} \) one has \( \frac{1}{\sqrt{3}} > \frac{1}{2} \).

No. 4
(1) Suppose \( x^3 \) was uniformly cont. on \( \mathbb{R} \). Then there would be a \( \delta > 0 \) s.t. \( \forall x, x_0 \in \mathbb{R} \)
\[
1 - x_0 | < \delta \implies 1 - x^3 \leq 1. \tag{3.12}
\]
Choose \( x = x_0 + \delta/2 \). Then
\[
| x^3 - x_0^3 | = | (x_0 + \delta/2)^3 - x_0^3 | = | \frac{3}{2} \delta x_0^2 + \frac{3}{4} \delta^2 x_0 + \frac{\delta^3}{8} |.
\]
Clearly, for \( x_0 \) large enough this will be \( \geq 1 \).

(2) By the Uniform Continuity Theorem \( \sin(x) \) is uniformly cont. on \( [0, 2\pi] \) i.e. for given \( \varepsilon > 0 \) there exists a \( \delta > 0 \) s.t. \( \forall x, x_0 \in [0, 2\pi] \)
\[
1 - x_0 | < \delta \implies |\sin(x) - \sin(x_0)| < \varepsilon.
\]
But since \( \sin(x) \) is periodic the above must hold true \( \forall x, x_0 \in \mathbb{R} \).
(iii) Suppose \( \sin \left( \frac{1}{x} \right) \) was unit con. on \((0, \infty)\). Then 
\[ \exists \epsilon > 0 \text{ s.t. } \forall x, x_0 > 0 \text{ one has} \]
\[ |x - x_0| < \delta \Rightarrow |\sin \left( \frac{1}{x} \right) - \sin \left( \frac{1}{x_0} \right)| < 1 \ (\epsilon) \]

Choose
\[ x = \frac{1}{2\pi \left( k + \frac{1}{4} \right)} \quad \text{and} \quad x_0 = \frac{1}{2\pi \left( k + \frac{3}{4} \right)} \]

where \( k \in \mathbb{Z} \) is so large that \( x, x_0 < \delta \). Then

\[ |x - x_0| < \delta \quad \text{but} \]
\[ |\sin \left( \frac{1}{x} \right) - \sin \left( \frac{1}{x_0} \right)| = |1 - (-1)| = 2 > 1 \]
For \( n \in \mathbb{Z}^+ \) let \( \Delta_n = \frac{b-a}{n} \). The Riemann sum of
\[
\int_a^b f(x) \, dx, \quad \frac{1}{r} \sum_{r=0}^{r=b} f\left( \frac{x_i}{r} \right) \Delta_n
\]
with respect to the partitions
\[
a, a+\Delta_n, a+2\Delta_n, \ldots, b \quad \text{and} \quad r_0, \ r_0+\Delta_n, \ r_0+2\Delta_n, \ldots, r_b
\]
is given by
\[
A_n = \sum_{i=0}^{n-1} f(a+i\Delta_n) \Delta_n \quad \text{and} \quad \frac{1}{r} \sum_{i=0}^{r} \left( \frac{r_0+i\Delta_n}{r} \right) \Delta_n = rB_n
\]
resp. But these are the same \( A_n = B_n \)! So,
\[
\lim_{n \to \infty} A_n = \lim_{n \to \infty} B_n = \frac{1}{r} \sum_{r=0}^{r=b} f(x) \, dx
\]

No. 6

Suppose there was a \( x_0 \in [a,b] \) s.t. \( f(x_0) > 0 \). Then let
\[
\varepsilon = \frac{|x_0|}{2} > 0 \quad \text{and let} \quad \delta > 0 \quad \text{be such that} \quad \forall x \in [a,b]
\]
\[
|\frac{x-x_0}{2}| < \delta \implies |f(x) - f(x_0)| < \varepsilon
\]
This implies that for all \( x \in [x_0-\delta, x_0+\delta] \) one has
\[
f(x) > f(x_0) - \varepsilon = f(x_0)/2 \quad \text{since} \quad f \geq 0 \quad \text{this gives}
\]
\[
\Theta = \int_a^b f(x) \, dx \geq \int_{x_0-\delta}^{x_0+\delta} \frac{f(x_0)}{2} \, dx = \frac{f(x_0)}{2} 2\delta > 0
\]
So, must have \( f(x_0) = 0 \quad \forall x_0 \in [a,b] \).
No. 7

Let $B > 0$ be such that $|f_1, f_2| \leq B$. Let $\varepsilon > 0$. Pick $\delta_1, \delta_2 > 0$ s.t. $\forall x, x_0 \in D$,

\[ |x - x_0| < \delta_1 \implies |f(x) - f(x_0)| < \frac{\varepsilon}{2B} , \]

\[ |x - x_0| < \delta_2 \implies |g(x) - g(x_0)| < \frac{\varepsilon}{2B} . \]

Let $\delta = \min(\delta_1, \delta_2) > 0$. Then $\forall x, x_0 \in D$,

\[ |x - x_0| < \delta \implies |f(x)g(x) - f(x_0)g(x_0)| \leq \]

\[ \leq |f(x)||g(x) - g(x_0)| + |g(x_0)||f(x) - f(x_0)| \]

\[ < B \cdot \frac{\varepsilon}{2B} + B \cdot \frac{\varepsilon}{2B} = \varepsilon . \]

So, $fg$ is uni. cont.

To see that boundedness is necessary in the assumptions, observe that

\[ f(x) = x, \quad g(x) = \sin(x) \]

are uni. cont. but $fg(x) = x\sin(x)$ is not.