Problem Set 6, Ma 001a-sec1, Fall 2016-17  
Due 4 pm Monday November 21st 

Remark: This homework set has 100 points.

1. (30 points) Determine the relative and absolute extrema and inflections points of the following functions and sketch their graphs:

(1) 
\[ f(x) = \frac{x^2 - 1}{x^2 - 4} \]

(2) 
\[ f(x) = (x^2 - 1)^{2/3} \]

(3) 
\[ f(x) = x + 2 \sin x \]

(4) 
\[ y = x^2 e^x \]

2. (15 points)

(1) Prove that among all rectangles that can be inscribed in a given circle, the square has the largest area.

(2) Find the largest possible area of a rectangle that can be inscribed in a semicircle of radius R, the lower base being on the diameter.

3. (20 points) Compute the derivative of the following functions:

(1) 
\[ f(x) = \int_0^{2x^2 - 4x + 5} \frac{1}{3 + \sin^6 t + t^2 + \sqrt{t}} dt \]

(2) 
\[ g(x) = \int_{x^2}^{2x^2 - 4x + 5} \frac{1}{3 + \sin^6 t + t^2 + \sqrt{t}} dt \]

(3) 
\[ h(x) = \int_0^{\cos^2 t} \frac{1}{3 + \sin^6 t + t^2 + \sqrt{t}} dt \]

4. (20 points) Calculate the following antiderivatives:

(1) 
\[ \int x^3 \cos x \, dx \]

(2) 
\[ \int \sin^2 x \, dx \]
(3) \[ \int \frac{x^3}{(x^4 + 16)^5} dx \]

5. (15 points) Find the area of the region bounded by

\[ y = (2 + \sin\left(\frac{x}{2}\right))^2 \cos\left(\frac{x}{2}\right), \]

the x-axis, and the lines \( x = 0 \) and \( x = \pi \).