1. (15 points) Assume $A$ and $B$ are non-empty bounded subsets of the real numbers $\mathbb{R}$ and let $-A = \{-x \mid x \in A\}$, $A + B = \{a + b \mid a \in A, b \in B\}$.

Show that

1. (1) $\inf A = -\sup(-A)$.
2. (2) $\sup(A + B) = \sup A + \sup B$.

2. (15 points) Let $f : \mathbb{R} \to \mathbb{R}$ be a polynomial of the form $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, where $n$ is a positive even integer. Prove that there is a $c \in \mathbb{R}$ such that $f(c) \leq f(x)$, for all $x \in \mathbb{R}$.

3. (10 points) Give an example of a function $f$ for which the following assertion is false: If $|f(x) - L| < \varepsilon$ when $0 < |x - a| < \delta$, then $|f(x) - L| < \frac{\varepsilon}{2}$ when $0 < |x - a| < \frac{\delta}{2}$.

4. (15 points) Prove that

1. (1) The function $f(x) = x^3$ is not uniformly continuous on the real line $\mathbb{R}$.
2. (2) The function $g(x) = \sin x$ is uniformly continuous on the real line $\mathbb{R}$.
3. (3) The function $h(x) = \sin\left(\frac{1}{x}\right)$ is not uniformly continuous on the interval $(0, 1)$.

5. (20 points) Using the definition of the integral using partitions, show that if $f$ is an integrable function on the interval $[a, b]$, then for any $r > 0$ we have:

$$\int_a^b f(x) \, dx = \frac{1}{r} \int_{ra}^{rb} f\left(\frac{x}{r}\right) \, dx.$$ 

6. (10 points) Suppose that $f$ is a continuous function on the interval $[a, b]$ and $f(x) \geq 0$ for all $x$ in $[a, b]$. If $\int_a^b f(x) \, dx = 0$ show that $f(x) = 0$ for all $x$ in $[a, b]$.

7. (15 points) Show that if $f$ and $g$ are uniformly continuous and bounded on $D \subset \mathbb{R}$, then $fg$ is uniformly continuous on $D$. By giving explicit examples of $f$ and $g$ show that this is not necessarily true if one of the functions is not bounded.