1. Let $G = (V, E)$ be a $k$-connected graph, and let $U, W$ be two disjoint subsets of $V$. We say that a $UW$-path is a simple path with one endpoint in $U$, the other endpoint in $W$, and containing no other points of $U$ and $W$ (that is, only the endpoints of the path are in $U$ and $W$). Prove that there exist $k$ $UW$-paths in $G$ such that every vertex of $V \setminus (U \cup W)$ is in at most one of the paths.

2. Let $G = (V, E)$ be a 2-connected graph. Prove that there exists an orientation of the edges of $E$ such that in the resulting directed graph there is a path from every vertex to every other vertex (that is, for every $v, u \in V$ there is a path directed path from $v$ to $u$ and another directed path from $u$ to $v$).

3. Let $G = (V, E)$ be a $k$-connected graph, for some $k \geq 2$, and let $|V| \geq 2k$. Prove that $G$ contains a (simple) cycle of length at least $2k$ (hint: consider the longest cycle in $G$ and how the additional vertices of $V$ are connected to it).

4. Let $G = (V, E)$ be a 2-connected graph with $|V| \geq 5$, and let $e \in E$. We define $G_1$ as the graph that is obtained by removing $e$ from $G$. We define $G_2$ as the graph that is obtained by contracting $e$ (as defined in Class 7). Prove that at least one of the graphs $G_1$ and $G_2$ is 2-connected.

5. Let $G = (V, E)$ be a graph that has $K_5$ as a topological minor, contains only vertices of degree three and four, and has the same number of vertices of degree three and of degree four. What is the minimum number of vertices in $G$? Prove your answer.