Due: Wednesday, March 2, 2016 at 9am.

All numbered problems are from Dummit and Foote, Third Ed.
All problems will be graded. Show all work to receive full credit.

Read Ch. 12 section 2 of the textbook.

• From section 12.2: problems 9 (only the first matrix), 10, 14, 17, 18.

• From section 12.3: problem 1.

• Let $V$ be a $n$-dimensional vector space over a field $F$. Let $X = (x_1, \ldots, x_n)$ be an ordered basis of $V$, and $\alpha \in \text{End}_F(V)$. Write $A = M_X(\alpha)$ for the matrix associated to $\alpha$ with respect to the basis $X$. Let $Y = (x_1, \ldots, x_m)$ and $Z = (x_{m+1}, \ldots, x_n)$ for some integer $m$, $0 < m < n$. Define $U = \langle Y \rangle$ and $W = \langle Z \rangle$.

Prove that

(1) $A = B \oplus C$ for some square matrices $B, C$ of size $m$ and $n - m$ respectively, if and only if $\alpha(U) \subset U$ and $\alpha(W) \subset W$, in which case $B = M_Y(\alpha|_U)$ and $C = M_Z(\alpha|_W)$.

(2) if $A = B \oplus C$ then the characteristic polynomial of $A$ is the product of those of $B$ and $C$ and the minimal polynomial of $A$ is the least common multiple of those of $B$ and $C$.

(3) if $A = B \oplus C$ prove that $A$ is diagonalizable if and only if $B$ and $C$ are diagonalizable.

(Warning: the statement "If $v \in V$ is an eigen-vector then $v$ is in $U$ or in $W$" is false.)