In the sequel, $V$ denotes a vector space defined over the field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ unless otherwise specified.

**Problem 1.** Read from the textbook: Chapter 5, Section 1.

**Problem 2 (20pts).** From the textbook: Chapter 4, Problem 2.6.

**Problem 3 (20pts).** Let $V = \mathbb{P}_3$ be the vector space of degree at most 3 polynomials in one variable $x$ (with complex coefficients). Let $T$ be the linear operator $T(f) = xf' + f''$. (You don’t need to check that $T$ is linear.)

(a) Calculate the eigenvalues of $T$.

(b) For each eigenvalue, find a basis of the corresponding eigenspace.

(c) Give a basis of $V$ for which $T$ is represented by a diagonal matrix.

**Problem 4 (20pts).** Let

$$A = \begin{pmatrix} 4 & 5 \\ 3 & 6 \end{pmatrix}.$$ 

Find a matrix $B$ satisfying $B^2 = A$.

**Problem 5 (20pts).** From the textbook: Chapter 5, Problem 1.5 (a), (b).

**Problem 6 (20pts).** From the textbook: Chapter 5, Problem 1.7 for the case when $\mathbb{F} = \mathbb{R}$. 