HOMEWORK 6

Due Tuesday, November 10 in class

Problem 1. (1991 A2) Let $A$ and $B$ be different $n \times n$ matrices with real entries. If $A^3 = B^3$ and $A^2 B = B^2 A$, can $A^2 + B^2$ be invertible?

Problem 2. (1995 B3) To each positive integer with $n^2$ decimal digits, we associate the determinant of the matrix obtained by writing the digits in order across the rows. For example, for $n = 2$, to the integer 8617 we associate $\det \begin{pmatrix} 8 & 6 \\ 1 & 7 \end{pmatrix} = 50$. Find, as a function of $n$, the sum of all the determinants associated with $n^2$-digit integers. (Leading digits are assumed to be nonzero; for example, for $n = 2$, there are 9000 determinants.)

Problem 3. (2002 A4) In Determinant Tic-Tac-Toe, Player 1 enters a 1 in an empty $3 \times 3$ matrix. Player 0 counters with a 0 in a vacant position, and play continues in turn until the $3 \times 3$ matrix is completed with five 1’s and four 0’s. Player 0 wins if the determinant is 0 and player 1 wins otherwise. Assuming both players pursue optimal strategies, who will win and how?

Problem 4. (2011 A4) For which positive integers $n$ is there an $n \times n$ matrix with integer entries such that every dot product of a row with itself is even, while every dot product of two different rows is odd?