1. (10 points) Find the equilibrium solution, draw a direction field, for the following equation

\[ y' = 2y - 3. \]

Then determine the behavior of \( y \) as \( t \to \infty \).

Solution: the equilibrium solution is

\[ y = 1.5. \]

The direction field is

As \( t \to \infty \), the solution with initial condition \( > 1.5 \) will go to \(+\infty\), and the solution with initial condition \( < 1.5 \) will go to \(-\infty\).

2. (10 points) Do the same analysis as in question [1] for

\[ y' = -1 - 2y. \]
Solution: the equilibrium solution is

\[ y = -0.5. \]

The direction field

As \( t \to \infty \), the solution will converge to the equilibrium solution.

3. (10 points) A spherical raindrop evaporates at a rate proportional to its surface area. Write a differential equation for the volume of the raindrop as a function of time.

Solution The mathematical formulation is

\[ \frac{dV}{dt} = -\kappa S \]

with \( S \) being surface area, and \( \kappa \) being a positive constant.

By the facts \( V = \frac{4}{3}\pi r^3 \) and \( S = 4\pi r^2 \), we have that

\[ S = \left(\frac{2}{3}\right)^{\frac{2}{3}} \pi \frac{1}{3} V^{\frac{2}{3}}. \]

Hence the final answer is

\[ \frac{dV}{dt} = -\kappa 6^{\frac{2}{3}} \pi \frac{1}{3} V^{\frac{2}{3}}. \]

4. (10 points) Solve the initial value problem

\[ \frac{dy}{dt} = -2y + 5, \]

\[ y(0) = 2. \]
Then determine the behavior of $y$ as $t \to \infty$.

solution The solution is

$$y(t) = \frac{5}{2} - \frac{1}{2} e^{-2t}. \quad (2)$$

and $y(t) \to \frac{5}{2}$ as $t \to \infty$.

5. (10 points) Solve the initial value problem

$$\frac{dy}{dt} = 2y + 10,$$

$y(0) = 2.$

Then determine the behavior of $y$ as $t \to \infty$.

solution The solution is

$$y(t) = -5 + 7e^{2t}. \quad (3)$$

and $y(t) \to +\infty$ as $t \to \infty$.

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