PROBLEM SET 8
Due on Friday, December 4

I. Conservative systems
1) Problem 34.1(ii). Also, find (approximately) the periods of small oscillations.

II. Dissipative systems
2-4) Problems 34.3(ii), 34.4, and 35.2

III. Lyapunov functions and stability of equilibrium solutions
5) Determine whether the critical point of the following system is stable or unstable:

\[
\dot{x} = -x^3 + 2y^3, \quad \dot{y} = -2xy^2.
\]
6) Show that the critical point (0, 0) of the following system is asymptotically stable:

\[
\dot{x} = 2y^3 - x^5, \quad \dot{y} = -x - y^3 + y^5.
\]

[Hint: construct a suitable Lyapunov function of the form \(ax^2 + y^4\).]

IV. Periodic orbits
7) Problem 36.1. Also, give an argument based on the dissipation of energy.
8) Problem 36.2. Also, draw the phase portrait of the system and show the limit cycle.

Final exam will be available 12/4 and will be due 12/10