PROBLEM SET 3
Due on Monday, October 19

I. Integration of 1st order equations

Solve the following equations:
1. \( \dot{x} + tx = 4t, \quad x(0) = 2 \)
2. \( y' = (x - y)/(x + y - 2) \)
3. \( x^2 y' + 2xy - y^3 = 0 \)
4. \( \dot{x} = t/(t^2 x + x^3), \quad (\text{Hint: } s = t^2) \)
5. \( y' = y^3/(1 - 2xy^2), \quad y(0) = 1 \)

II. Application of 1-st order ODEs

6. Find all differentiable functions \( f(x) \) satisfying the functional equation
\[
 f(x + y) \equiv \frac{f(x) + f(y)}{1 - f(x)f(y)}
\]
on some interval containing zero.

7. Suppose that a truck starts at a point \((a,0)\) on a desert and moves north (the direction of the \(y\)-axis). Let a police car start at a point \((0,0)\) and pursue the truck across the desert so that the velocity vector of the police car always points towards the truck. Assume that the police car travels twice as fast as the pursued. Find the point at which the police captures the truck.

III. Reduction of order

8. Solve the equation \( y'' + (y')^2 = 2e^{-y} \).