1. For any real vector bundle $E$ of rank $n$ over a CW complex $X$, the complexification $E_C$ over $X$ is the complex vector bundle with fibers $(E_C)_x = E_x \otimes \mathbb{C}$ over $x \in X$. Denote by $(E_C)_R$ the underlying real vector bundle of rank $2n$ by forgetting the complex structure. Show that there is an isomorphism:

$$(E_C)_R \cong E \oplus E.$$

2. Let $X$ be a finite CW complex. For any complex vector bundle $E$ over $X$ of rank $n$, define the determinant bundle $\det(E)$ over $X$ to be the line bundle which is the top-dimensional exterior power $\wedge^n E$ of $E$. Show that the following bundle isomorphisms exist:

1. $\det(E') \otimes \det(E/E') \cong \det(E)$, for any sub-bundle $E'$ of $E$;
2. $\det(E^*) \cong \det(E)^*$, where $*$ denotes the dual.

3. Show that the projection $V_n(\mathbb{R}^k) \to G_n(\mathbb{R}^k)$ is a fiber bundle with fiber $O(n)$ by showing that it is the orthonormal $n$-frame bundle associated to the vector bundle $E_n(\mathbb{R}^k) \to G_n(\mathbb{R}^k)$. [VBKT p. 37, Ex. 1]