1. Let $A$ be a central simple algebra over the field $F$ and $\alpha : A \to A$ an algebra automorphism (a ring automorphism which is the identity on $F$). Show that $\alpha$ is inner (i.e. $\alpha(x) = axa^{-1}$ for some $a \in A^\times$). This is called the Skolem-Noether theorem. Hint: Use the isomorphism $A \otimes_F A^{\text{op}} \cong \text{End}_F(A)$ to write $\alpha(x) = \sum e_i x a_i$ for some $F$-basis $e_i$ of $A$.

2. Show that $A \cong A^{\text{op}}$ where $A = \begin{pmatrix} a & b \\ \overline{c} & d \end{pmatrix}$ is the quaternion algebra from the last problem set. Deduce that $A$ represents an element of order dividing 2 in the Brauer group of $F$.

3. Show that if $C$ and $D$ are division rings with center $F$ of coprime dimension (i.e. $\gcd(\dim_F D, \dim_F C) = 1$) then $C \otimes_F D$ is a division ring (hint: argue with dimensions of appropriate modules over $C$ and $D$).

4. Let $C_n'$ (resp. $C_n$) be the Clifford algebra associated to the standard (resp. the negative of the standard) Euclidean inner product on $\mathbb{R}^n$ by the general construction of Lang Ch. XIX, §4.
   a) Show that the map $\psi : \mathbb{R}^{n+2} \to C_n' \otimes_{\mathbb{R}} C_2$ defined by
      $$\psi(e_\nu) = \begin{cases} e_{\nu-2} \otimes e_1 e_2 & 3 \leq \nu \leq n + 2 \\ 1 \otimes e_\nu & \nu = 1, 2 \end{cases}$$
      induces an isomorphism $C_{n+2} \cong C_n' \otimes_{\mathbb{R}} C_2$. Prove similarly $C_{n+2}' \cong C_n \otimes_{\mathbb{R}} C_2'$. Use the isomorphisms
      $$C_0 = C_0' = \mathbb{R}; \quad C_1 = C_1' = \mathbb{H}; \quad C_2 = \mathbb{H}; \quad C_2' = M_2(\mathbb{R})$$
      to compute $C_i$ and $C_i'$ for $i = 3, 4, 5, 6, 7$ and show that $C_{i+8} \cong C_i \otimes_{\mathbb{R}} M_{16}(\mathbb{R})$ and $C_{i+8}' \cong C_i' \otimes_{\mathbb{R}} M_{16}(\mathbb{R})$. 
      