Homework 7: Due Tuesday May 19, 1pm

1. (15 pts) Prove the theorem of generalization on constants. Suppose that L is a language, ϕ is a formula in L, and S is a set of formulas in L. Now suppose we enlarge our language L to $L \cup \{c\}$ by adding a new constant symbol c, and in this language, $S \vdash \phi_c^x$, where ϕ_c^x is the formula with each free instance of x replaced by c. (Note that no formula of S includes the constant c). Then show $S \vdash \forall x \phi$.

2. (15 pts) Prove the soundness theorem for first order logic: Suppose L is a language, S is a set of sentences in the language L, and ϕ is a sentence in the language L. Show if $S \vdash \phi$, then $S \vDash \phi$.

3. (15 pts, no collab) Prove the compactness theorem for first order logic. Suppose L is a language, S is a set of sentences in L, and ϕ is a sentence in L. Prove if $S \vDash \phi$, then there is a finite subset $S_0 \subseteq S$ such that $S_0 \vDash \phi$. [Hint: First use the completeness theorem. Then use the fact that proofs are finite.]

4. (15 pts) Suppose that T' is a consistent Henkin theory in the language L', and suppose $\{\theta_1, \theta_2, \ldots\}$ is a list of all formulas in the language L'. Then show that there is a theory $T'' \supseteq T'$ such that T'' is a *complete* consistent Henkin theory. (This essentially finishes our proof of Lemma 11.3 in the notes.)