1. (15 pts) Prove the theorem of generalization on constants. Suppose that $L$ is a language, $\phi$ is a formula in $L$, and $S$ is a set of formulas in $L$. Now suppose we enlarge our language $L$ to $L \cup \{c\}$ by adding a new constant symbol $c$, and in this language, $S \vdash \phi^c$, where $\phi^c$ is the formula with each free instance of $x$ replaced by $c$. (Note that no formula of $S$ includes the constant $c$). Then show $S \vdash \forall x \phi$.

2. (15 pts) Prove the soundness theorem for first order logic: Suppose $L$ is a language, $S$ is a set of sentences in the language $L$, and $\phi$ is a sentence in the language $L$. Show if $S \vdash \phi$, then $S \models \phi$.

3. (15 pts, no collab) Prove the compactness theorem for first order logic. Suppose $L$ is a language, $S$ is a set of sentences in $L$, and $\phi$ is a sentence in $L$. Prove if $S \models \phi$, then there is a finite subset $S_0 \subseteq S$ such that $S_0 \models \phi$. [Hint: First use the completeness theorem. Then use the fact that proofs are finite.]

4. (15 pts) Suppose that $T'$ is a consistent Henkin theory in the language $L'$, and suppose $\{\theta_1, \theta_2, \ldots\}$ is a list of all formulas in the language $L'$. Then show that there is a theory $T'' \supseteq T'$ such that $T''$ is a complete consistent Henkin theory. (This essentially finishes our proof of Lemma 11.3 in the notes.)