

## Homework 3: Due Tuesday April 22, 1pm

1. (20 pts). (Review Problem). Use the Compactness Theorem for Propositional Logic (either version) to prove König's Lemma. In class we did the converse, so this establishes that the two principles are equivalent.
2. (10 pts) Prove the correctness of the algorithm given in class for efficiently converting a formula  $\phi$  into a formula  $\psi$  in CNF which is satisfiable iff  $\phi$  is.
3. (8 pts) Using resolution, show that the set of clauses

$$\begin{aligned} & \{ \{p, q, r\}, \{\neg p, \neg q, \neg r\}, \{\neg p, q, s\}, \{p, \neg q, \neg s\}, \\ & \qquad \qquad \qquad \{p, \neg r, s\}, \{\neg p, r, \neg s\}, \{\neg q, r, s\}, \{q, \neg r, \neg s\} \} \end{aligned}$$

is unsatisfiable.

4. (15 pts) (No collaboration) Using our Hilbert-style formal proof system, show that:
  - (a) From  $\{\neg p \rightarrow q, \neg p \rightarrow \neg q, p \rightarrow (r \rightarrow s), p \rightarrow r\}$  there is a formal proof of  $s$ .
  - (b) For every formula  $\phi$ , there is a formal proof of  $\neg\neg\phi \rightarrow \phi$ .