Homework Set 2, Math1c Analytical, Spring 2015

Due Monday, April 13 at 10:00 am

Do all five problems. Give details; part of what one must learn - in the analytical track at least - is precise logical reasoning with no gaps. You may use any result from the class notes or the text (except exercises) but you must give a precise reference. Each student must write up her or his solution individually, even if the solution resulted from collaboration. Section numbers below refer to Apostol’s book.

Week 2, 4/6-4/10

Reading: 8.6-8.13; Ch 2.1, 2.2 from class notes

Problems:

1) 8.9: 9, 16

2) 8.14: 5 (Note: “in a direction parallel to the z-axis” means “in the direction of the positive z-axis.”)

3) 8.14: 8

4) Consider the scalar field \( f : \mathbb{R}^2 \to \mathbb{R} \) defined by

\[
f(x, y) = (x^2 + y^2) \sin\left(\frac{1}{\sqrt{x^2 + y^2}}\right); \text{ if } (x, y) \neq (0, 0)
\]

and \( f(0, 0) = 0 \). Put \( a = (0, 0) \).

a) Show that \( f \) is differentiable at \( a \). Compute the matrix of the total derivative of \( f \) at \( a \).

b) Show that neither \( \frac{\partial f}{\partial x} \) nor \( \frac{\partial f}{\partial y} \) is continuous at \( a \).

5) Is the function

\[
f(x, y) = \begin{cases} 
\frac{xy}{\sqrt{x^2 + y^2}}, & \text{if } (x, y) \neq (0, 0), \\
0, & \text{if } (x, y) = (0, 0), 
\end{cases}
\]

differentiable at \( (0, 0) \)? Justify your answer.