Midterm Exam, Math 1c Analytical, Spring 2015

Time limit is 4 hours in one sitting. The solutions are due on Monday, May 4, noon in the slots outside the copier room Sloan 255.

You may use Apostol’s book, posted class notes, your own class notes, your returned homework and the posted solutions to the homework but no other tools. Be careful in justifying the steps in your arguments and quote results from the book or the class notes as you go along.

Finally: Use a blue book and put your section number and your name on the exam!
1) Let $p(x)$ be a polynomial of odd degree in one variable with real coefficients. For each closed interval $[a, b], a < b, a, b \in \mathbb{R}$, show that the set $\{x \in \mathbb{R}: p(x) \in [a, b]\}$ is compact.  

20 points

2) Let $\alpha \in \mathbb{R}$ and define $f: \mathbb{R}^3 \to \mathbb{R}$ by

$$f(x, y, z) = \begin{cases} \frac{xyz^2}{(x^2+y^2+z^2)\alpha}, & \text{if } (x, y, z) \neq (0, 0, 0), \\ 0, & \text{if } (x, y, z) = (0, 0, 0). \end{cases}$$

Show that $f$ is differentiable at $(0, 0, 0)$ if and only if $\alpha < \frac{3}{2}$.  

20 points

3) Let $g: \mathbb{R} \to \mathbb{R}, g(x) = ax + b$, be a linear function, and let $h: \mathbb{R} \to \mathbb{R}, f: \mathbb{R} \to \mathbb{R}$ be in $C^2$. Put $u(x, y) = f(g(x) + h(y))$. Show that

$$\frac{\partial u}{\partial x} \cdot \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial y} \cdot \frac{\partial^2 u}{\partial x^2}.$$  

20 points

4) Consider the surface

$$S = \{(x, y, z) \in \mathbb{R}^3: x \geq 0, y \geq 0, z \geq 0, xyz = \alpha\},$$

where $\alpha > 0$. Show that there is a unique point in $S$ of minimum distance from the origin and calculate its coordinates.  

20 points

5) Let $s > 0$. Show that among the 3-dimensional rectangular boxes of surface area $s$, there is a unique one of maximum volume and calculate its dimensions.  

20 points