Read from the textbook: Chapter 2, Sections 9–15.

You can collaborate on the problems as long as you write up all solutions in your own words and understand those solutions.

(1) (9 pts) Let $V$ be a finite dimensional vector space. For $U \leq V$ define
\[ \text{codim } U = \dim V - \dim U. \]
Let $U_i, i = 1, \ldots, r$, be subspaces of $V$ and $W = \bigcap U_i$. Prove
(a) $\text{codim } W \leq \sum_i \text{codim } U_i$,
(b) if $\sum_i \text{codim } U_i < \dim V$ then $W \neq 0$.

(2) (9 pts) From Ch.2.8 in Apostol: Problem 28.

(3) (6 pts) In each of the following, a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is described. Find dimensions and explicit bases of the Null space and Range of $T$. If $T$ is invertible, compute $T^{-1}$.
(a) $T(x, y, z) = (x + y, x + z, y + x)$,
(b) $T(x, y, z) = (x, x + y, y)$,
(c) $T(x, y, z) = (x + z, 0, x + y)$,
(d) $T(x, y, z) = (x, x - y, z - y)$,
(e) $T(x, y, z) = (x, y + z, y - z)$,
(f) $T(x, y, z) = (x, x + y, x + y + z)$.