The following is a list of basic questions, rephrased in different contexts.

**Vector Spaces**
*Notions:* subspaces, linear combinations, minimal linear subspace spanned by a collection of vectors, intersection, sum.

Let $S = \{v_1, \ldots, v_n\}$ be a collection of vectors of $V$:
1. Is $S$ linearly independent?
2. Is $S$ a generating set?
3. Is $S$ a basis of $V$?
4. Compute a basis of $L(S)$.
5. Given $w$ in $V$: is $w \in L(S)$? If possible, write $w$ as a linear combination of the vectors in $S$. Are the coefficients unique?
6. If they exist, exhibit non-trivial linear combinations among the vectors in $S$.

**Linear transformations**
*Notions:* range, nullspace, composition, inverses, preimages.

Let $f : V \to W$ a linear transformation:
1. Is $f$ injective?
2. Is $f$ surjective?
3. Is $f$ bijective?
4. Compute the inverse of $f$.
5. Compute a basis of the range of $f$.
6. Given $w \in W$: is $w$ in the range? If it exists, find $v \in V$ such that $f(v) = w$.
7. Compute a basis of the nullspace of $f$.

**Matrices and Linear systems**
*Notions:* multiplications, inverses, elementary matrices, REF, homogenous and non-homogenous linear systems.

Let $Ax = b$ be a linear system.
1. If a solution exists, is it unique?
2. Does a solution exists for any $b$?
3. Does a solution exists and is unique for any $b$?
4. Compute the inverse of $A$.
5. For which $b$ does the linear system admits solutions?
7. Solve the associated homogenous linear system.

**Determinants and minors**
*Notions:* singular matrices, rank of a matrix, nullity of a matrix.

Let $Ax = b$ be a linear system, $n$ variable, $m$ equations.
1. Is the rank equal to $n$?
2. Is the rank equal to $m$?
3. Is the matrix non-singular? A.k.a. is the determinant non-zero?
4. Compute the inverse of $A$.
5. For which $b$ is the rank of $A$ equal to the rank of the completed matrix $(A|b)$?
(6) Given $b$: compute the ranks of $A$ and $(A|b)$.

**Euclidean spaces**

*Notions:* inner product, distance, orthogonality, isometries, Gramm-Schmidt, projections, best approximation.

Let $S$ be a collection of vectors of an euclidean space $V$:

1. Is $S$ orthogonal?
2. Find an orthogonal/orthonormal basis $B$ of $L(S)$.
3. Given $w \in V$: is $w \in L(S)$? If possible write $w$ as a linear combination of the vectors in $B$. If not, compute the distance of $w$ from $U$.
4. Compute the orthogonal complement of $L(S)$. A.k.a. extend $B$ to an orthogonal/orthonormal basis of $V$.

**Eigenvalues and eigenvectors**

*Notions:* characteristic polynomial, algebraic multiplicity and geometric multiplicity of eigenvectors. Diagonalization. Similitude.

Let $V$ be a vector space and $T : V \to V$ a linear transformation

- Compute the characteristic polynomial of $T$. A.k.a. compute the eigenvalues of $T$.
- Is $T$ diagonalizable? A.k.a. does $V$ has a basis consisting of eigenvectors of $T$.
- If possible, find one / all diagonal matrices representing $T$ with respect to a basis (of eigenvectors) of $V$.
- Compute all eigenspaces of $T$. If possible, find a basis of $V$ consisting of eigenvectors.
- For $V$ is an euclidean space: if possible, find an orthonormal basis of $V$ consisting of eigenvectors.