1) Let \( \{ \varphi_e \} \) be an acceptable effective enumeration of the partial recursive functions. Let \( h \) be a total recursive function. Show that there are infinitely many \( n \) such that 
\[
\varphi_{h(n)} = \varphi_n.
\]
Conclude that no acceptable effective enumeration of the partial recursive functions is 1–1.

2) Show that if \( \{ \varphi_e \} \) is an acceptable effective enumeration of the partial recursive functions, then there is a total recursive function \( p \) such that \( \varphi_{p(i)}(x) = \varphi_i(p(i), x) \). Similarly, show that there is a total recursive function \( q \) such that for any \( i \) for which \( \varphi_i \) is total, we have \( \varphi_{q(i)} = \varphi_{\varphi_i(q(i))} \).

3) Let \( \{ \varphi_e \} \) be an acceptable effective enumeration of the partial recursive functions. Show that there are \( m \neq n \) such that \( \varphi_m(x) = n \) and \( \varphi_n(x) = m \).

4\*) Let \( \{ \varphi_e \} \) be an acceptable effective enumeration of the partial recursive functions. Let \( W_e = \text{domain}(\varphi_e) \). Then \( \{ W_e \} \) is an effective enumeration of the r.e. sets, i.e., each \( W_e \) is r.e., for each r.e. set \( A \) there is \( e \) such that \( A = W_e \) and the relation \( W(e, x) \Leftrightarrow x \in W_e \Leftrightarrow \varphi(e, x) \downarrow \) is r.e. Use 2) above, to show that for any total recursive function \( h(x, y) \), there is a total recursive function \( s(x) \) such that \( W_{s(x)} = W_{h(x,y)} \).

Call a set \( P \) productive if there is a partial recursive function \( f \) such that for any \( e \), if \( W_e \subseteq P \), then \( f(e) \downarrow \) and \( f(e) \in P \setminus W_e \). Show that if \( P \) is productive, then for any co-r.e. set \( A \) (i.e., a set whose complement is r.e.), there is a total recursive function \( g \) such that \( x \in A \Leftrightarrow g(x) \in P \).

**Hint.** Show that there is a total recursive function \( s \) such that if \( x \in A \) then \( W_{s(x)} = \emptyset \) and if \( x \notin A \), then \( f(s(x)) \downarrow \) and \( W_{s(x)} = \{ f(s(x)) \} \).

**Note:** Homework rules are posted in the Ma/CS 117a web page.
Each problem is worth 25 points.