Homework 9 *

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Problem 1. Let A be a real 4×4 skew-symmetric matrix (i.e., $A^t = -A$, where A^t is the transpose of A). Prove that $\det A \geq 0$.

Problem 2. Alice and Bob play a game in which they take turn filling entries of an initially empty 2008×2008 array. Alice plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all the entries are filled. Alice wins if the determinant of the resulting matrix is nonzero; Bob wins if it is zero. Which player has a winning strategy?

Problem 3. Let A and B be different $n \times n$ matrices with real entries. If

$$A^3 = B^3 \text{ and } A^2 B = B^2 A,$$

can $A^2 + B^2$ be invertible?

Problem 4. Let d_n be the determinant of the $n \times n$ matrix whose entries, from the left to the right and from top to bottom, are $\cos 1, \cos 2, \ldots, \cos n^2$ (the argument of cos is in radians, not degrees). For example,

$$d_3 = \det \begin{pmatrix} \cos 1 & \cos 2 & \cos 3 \\ \cos 4 & \cos 5 & \cos 6 \\ \cos 7 & \cos 8 & \cos 9 \end{pmatrix}$$

Evaluate $\lim_{n \to \infty} d_n$.

*Due on 12/02/2013, in class.