Homework 8 *

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Problem 1. Let S be a set of real numbers which is closed under multiplication (i.e., if $a, b \in S$ then $ab \in S$). Let T and U be disjoint subsets of S whose union is S. Given that the product of any three elements of T is in T and that the product of any three elements of U is in U, show that at least one of the two subsets T, U is closed under multiplication.

Problem 2. Consider a set Σ with a binary operation $\star$. If

$$(x \star y) \star x = y, \forall\ x, y \in \Sigma,$$

prove that

$$x \star (y \star x) = y, \forall\ x, y \in \Sigma.$$

Problem 3. Let S be a nonempty set with an *associative* binary operation such that

$$xy = xz \text{ implies } y = z$$

and

$$yx = zx \text{ implies } y = z.$$

Assume that for every $a \in S$ the set $\{a^n : n = 1, 2, 3, \dots\}$ is finite. Must S be a group?

Problem 4. Show that a finite group can not be the union of two of its *proper*[1] subgroups.
(Optional: Does the statement remain true if "two" is replaced by "three"?)

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*Due on 11/25/2014, in class.
[1]Recall that a subgroup H of a group G is proper iff $H \neq G$.